Mathematics Lesson Plan (for Grade Five)

For the lesson on Friday, August 10, 2001
At Brewer Island School, San Mateo, CA
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1. Title of the Lesson: How many edges do I need to cut in order to open a cube?

2. Goal:

a. Deepen students' understanding of three-dimensional geometric objects through problem solving activities.
b. Help students become good problem solvers by providing a challenging open-ended problem.
   i. Encourage students to use their existing knowledge to solve a challenging problem.
   ii. Encourage students to find common properties and relationships among various patterns by comparing peers' solutions in order to find a solution to the problem.
   iii. Encourage students to consider their solutions from a different perspective, so that they can make reasoned conjectures.
c. Provide students with opportunities to find the importance of working with peers to deepen their understanding of mathematics.

3. Relationship of the Lesson to the Mathematics Content Standards for California Public Schools K-Grade12.

Grade Two
Students identify and describe the attributes (the number and shape of faces, edges, and vertices) of common figures in the plane and of common objects in space (including cube, rectangular prism, sphere, and pyramid).

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Grade Three
Identify, describe, and classify common three-dimensional geometric objects (including cube, rectangular solid, sphere, prism, pyramid, cone, cylinder).
Identify common solid objects that are the components needed to make a more complex solid object.

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Grade Four
Visualize, describe, and make models of geometric solids in terms of the number and shape of faces, edges, and vertices; interpret two-dimensional representations of three-dimensional objects; and draw patterns (of faces) for a solid that, when cut and folded, will make a model of the solid.

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This Problem Solving Lesson

Grade Five
Construct a cube and rectangular box from two-dimensional patterns and use those patterns to compute the surface area for those objects.
Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

4. Instruction of the Lesson

The lesson is designed to provide students with an opportunity to use their understanding of geometric objects, developed though previous mathematics lessons, in order to solve a problem that students otherwise might not be able to solve, because no routine path is apparent.

According to the *Mathematics Content Standards for California Public Schools K-Grade12*, in grade three students are introduced to the cube as one common three-dimensional geometric object. In grade four, students experience visualizing, describing, and making models of geometric solids. Through these learning experiences, students develop their understanding of basic geometric solids such as the cube. These experiences include:

- Describing a geometric solid in terms of the number of faces, edges, and vertices.
- Interpreting two-dimensional representations of three-dimensional objects.
- Drawing patterns of faces for a solid that, when cut and folded, will make a model of the solid.

In today's lesson, students are expected to solve the following problem using the above learning experiences.

> How many edges of a cube do you need to cut in order to open a cube completely and make a net? Find the least number of edges that need to be cut.

One reason why I have chosen this problem is that it provides students with an opportunity to extend their problem solving strategies. In order to solve this problem, first, students may actually cut and open cube models so that they can find how many edges they need to cut to open the cube. Next, students are expected to establish a conjecture that the least number of edges might be seven; however, the answer cannot be finalized by opening only one or two cube models. Since there are eleven different ways to open a cube by cutting seven edges, which means there are eleven different patterns of nets, students will have the opportunity to compare and discuss with peers to find general properties and relationships among the eleven nets, and this will lead students to establish a conjecture. This series of problem solving activities will help students develop their problem solving strategies. Students are expected to develop the following strategies described in the *Mathematics Content Standards for California Public Schools Grade 5*:

- Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning (Mathematical Reasoning 2.3).
- Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work (Mathematical Reasoning 2.4).
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Another reason for choosing this problem is it provides students with an opportunity to use what they have learned through the end of grade four. This lesson will play an important role in bridging students' previous understanding of the material with the California Standards goal for the new grade level. Since students will be expected to construct cubes and rectangular boxes from two-dimensional patterns and use the patterns to compute the surface

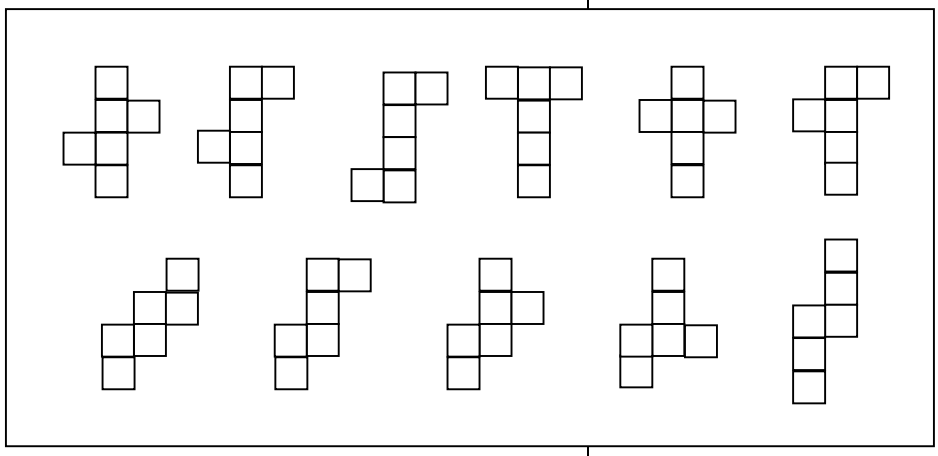
areas of these objects in grade five, this problem will establish an opportunity not only to develop students' problem solving strategies, but also to make a connection from the contents that students have learned in prior years to the new content they will learn in grade five.

5. Lesson Procedure

<table>
<tr><th>Learning Activities
Teacher's Questions and Expected Student Reactions</th><th>Teacher's Support</th><th>Points of Evaluation</th></tr>
<tr><td>1. Introduction to the Problem

How many edges of a cube do you need to cut in order to open a cube completely and make a net? Find the least number of edges that need to be cut.</td><td>Ask students to tell what they have learned about cube by showing a model of cube.

Show students how to cut and open a cube by using a model if necessary.</td><td>Do the students recall the properties of cube?

Do students understand the problem?</td></tr>
<tr><td>2. Individual Problem Solving
Find how many edges need to be cut to open a cube by opening several cube models.
  </td><td>Encourage students to find as many different ways to open a cube as possible.</td><td>Does each student find more than two ways to open a cube?</td></tr>
<tr><td>3. Comparing and Discussing
(1) Help students form a conjecture that the least number of edges might be seven.

(2) Facilitate students' discussion about their conjecture. Five edges must remain attached in order to make a cube turn into a two-dimensional pattern from a cube. If six edges remain attached, a cube cannot turn into a two-dimensional pattern.

(3) Help students discover a relationship between the number of edges that a cube has and the number of edges that connect six faces in each net.</td><td>Help students to discover that all eleven nets share common properties:
• Each net has six faces.
• Six faces are connected by five edges.
•Seven edges should be cut to open a cube and to make a net.

[12: number of edges of a cube] – [5: number of the edges remaining attached after opening a cube] = [7: number of edges to be cut]</td><td>Does each student find out that all the eleven nets share common properties?</td></tr>
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4. **Find the solution to the problem** Help students understand that they need to cut at least seven edges in order to open a cube.		
5. **Summing up** (1) Using a blackboard writing, review what students learned through the lesson. (2) Ask students to write a journal entry of what they learned through the lesson.		

6. Evaluation

a. Were the students able to find several ways to open a cube and find out how many edges needed to be cut?

b. Were the students able to compare the eleven patterns of nets and find general properties and relationships among the nets to establish a conjecture?

c. Were students able to review what they learned through the lesson and write about it in their journal?