Lesson Research Proposal for Third Grade Division

1. **Title of the Lesson: Quotative Division - How many kids will get 3 cookies each if there are 12 to start?**

2. **Brief description of the lesson**

Students will explore a new kind of division word problem using the models and tools developed in the multiplication unit to solve for the missing factor.

3. **Research Theme**

Research Theme:

Deepening Conceptual Understanding through a problem solving approach with a specific focus on the operations and algebraic thinking.

Our Theory of Action:

If teachers apply a Teaching Through Problem Solving based approach (TTP), then students will deepen their conceptual understanding of mathematics. This will result in students being able to apply their learning in multiple contexts and to justify their thinking by utilizing or making connections between multiple representations. Conceptual understanding will also support procedural fluency, as students make connections and see patterns between content. Students will understand and apply algorithms effectively as a result of this conceptual foundation. In order for this type of learning approach to occur, classrooms must be socially and emotionally safe where mistakes are valued. Students must work collaboratively to develop skills of productive struggle, problem solving, communication and stamina. Students must develop a growth mindset. Student discussion and journals serve as a way to formatively assess students' mathematical understanding.

4. **Goals of the Unit**

The purpose of this unit is to develop students' conceptual understanding of division. Students come to recognize scenarios involving "equal sharing" as situations that can be represented mathematically with division. Students explore both partitive (when the size of the groups is unknown) and quotative (when the number of groups is unknown) equal sharing situations and recognize that both types of situations can be represented with division.

Students also connect their new learning about division to their prior learning about multiplication. As students see that, as in multiplication, a division equation involves a total, a number of groups, and a size of the groups, they begin to understand the relationship between the two operations. They begin to recognize multiplication as a tool for solving division problems, and in so doing they recognize division as a "missing factor" problem. The inverse relationship between multiplication and division is reinforced through these calculations.

Over the course of the unit, students move from concrete representations of division ("dealing out" counters into equal-sized piles) to more representational (models such as tape diagrams and arrays) and symbolic (equations). Finally, students apply their learning to an assortment of division and multiplication problems, learning to distinguish division and multiplication scenarios in word problems, and to multiply and divide within 100.

5. Goals of the Lesson:

Students will further their understanding of division with a new situation - quotative division. Students will analyze the question in order to recognize the situation in the story as being different from the partitive situations they have experienced. Students build on work in previous multiplication and division lessons towards understanding the connection between their equations and the problem situation, including understanding what each number in an equation represents.

6. Relationship of the Unit to the Standards

Related prior learning standards	Learning standards for this unit	Related later learning standards
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. CCSS.MATH.CONTENT.K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal	CCSS.MATH.CONTENT.3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8*. CCSS.MATH.CONTENT.3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1	CCSS.MATH.CONTENT.4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. CCSS.MATH.CONTENT.

explanations, expressions, or equations. CCSS.MATH.CONTENT.K.OA.A.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. CCSS.MATH.CONTENT.K.OA.A.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **Work with equal groups of objects to gain foundations for multiplication.** CCSS.MATH.CONTENT.2.OA.C.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even	CCSS.MATH.CONTENT.3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* Understand properties of multiplication and the relationship between multiplication and division. CCSS.MATH.CONTENT.3.OA.B.5 Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* CCSS.MATH.CONTENT.3.OA.B.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*. Multiply and divide within 100. CCSS.MATH.CONTENT.3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. Solve problems involving the four operations, and identify and explain patterns in arithmetic. CCSS.MATH.CONTENT.3.OA.D.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3	4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. CCSS.MATH.CONTENT.4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division*. CCSS.MATH.CONTENT.4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations,

number as a sum of two equal addends. CCSS.MATH.CONTENT.2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	CCSS.MATH.CONTENT.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.	rectangular arrays, and/or area models. CCSS.MATH.CONTENT.5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used .

7. Background and Rationale

This year the third grade team has made some significant shifts in how we teach multiplication and division. Last year we used Eureka Math as the basis for our instruction, whereas this year we have shifted to a Teaching Through Problem-Solving (TTP) approach. We have also based our multiplication and division units on Japanese curriculum and on our work with Dr. Takahashi during the OUSD Summer Lesson Study Institute and his presentation at the Mills College Children's School on second and third grade math instruction.

A big difference in our approach to multiplication and division this year is in the sequence in which we introduce the operations. Based on our work this summer and on the Japanese curriculum, we decided to spend a significant amount of time early in the year focusing only on multiplication, before moving on to division and finally a combination of the two operations. In contrast, the Eureka curriculum we used last year teaches division alongside multiplication, introducing division almost immediately (in the fourth lesson of the first unit). It also introduces quotative and partitive division very close together (lessons 4 and 5) and then alongside each other for the rest of the first and third units without much distinction.

This year we made a conscious decision to space these learnings out. We wanted to give students a solid amount of time to explore and understand the new brand operation of multiplication conceptually before moving on to a second new operation with division. In this current unit plan, we also intentionally left space between introducing the two types of division- partitive and quotative. Our aim is to help kids really understand the two different problem situations, as well as their similarities and differences.

We believe that these planning decisions may be able to help our students with a major misunderstanding we have seen in multiplication: students are quick to choose an operation and write an equation, but struggle to interpret their results in a real world context. For example, in a multiplication problem "There are 3 boxes with 2 kittens in each box; how many kittens are there in total?" students might know to set up the equation 3 x 2 = 6, but be unable to answer the question "6 whats?" Many refer to 6 as the "answer," but fewer are able to name it as the "total." Identifying what each number represents is especially tricky with division, where the "answer" to a division problem could be either the number of groups or the size of the groups. Students face even more confusion when they try to solve division as a missing factor problem. For example, in the problem "There are 6 kittens in total, and there are 2 in each box, how many boxes are there?" Students might set up the equation 3 x 2 = 6, but then lose track of which number was the unknown and give the answer as 6 (in most previous experiences, the number to the right of the equal sign is the "answer," but here it is not). A major emphasis in our units, then, has been to get kids to realize the importance of understanding what each number represents.

Another very big change we have made this year in our teaching of this topic is to move completely to a Teaching Through Problem Solving approach. This allowed us to base our instruction on students' own ideas and to move the unit forward in response to their understandings and misunderstandings. One big shift that this allowed us was to be able to spend more time in the beginning of the multiplication and division units on models that are more concrete (manipulatives, equal group drawings) and to introduce more representational or abstract models (tape diagrams, complex equations) as students gain more understanding. This stands in stark contrast to Eureka, in which students are very quickly moved towards more abstract and representational models, which are taught through direct instruction. The curriculum then demands that certain problems be solved with certain models, regardless of whether they are the most appropriate for the situation or whether the students understand them.

In contrast, in the TTP approach, we've made the space for students to use strategies they are comfortable with, to adopt new strategies as they are introduced by classmates and as they make sense to the students. And we have also been asking the students themselves to consider which method they think is most appropriate for the situation (2 x 3 is fine for an equal groups drawing, but 7 x 8 demands a more efficient solution). We believe this approach can build student's conceptual understanding as well as student agency.

So far in teaching this unit, Ms. Cook has seen some exciting mathematical thinking in her kids. At least ½ are comfortable using the distributive property effectively. Some who were confused mid-module seem to be skip counting and understanding the concept more.

The topic we have chosen for our research lesson is one on quotitive division. At this point in the unit, students will already have been introduced to partitive division. We chose this lesson in part because we think this type of division, where the number of groups is unknown, might be more challenging for our students. This type of division is more abstract and does not lend itself to "dealing" objects into groups (fair share division). In multiplication we always started knowing the number of groups and then could draw the number of things *in* each group. This type of division requires students to start with the number *in* each group, which may feel less comfortable. Because this is the very beginning of their experience with division, we are eager to see how they make sense of the problem and how they attempt to solve it.

8. Research and *Kyozaikenkyu*

In studying the Common Core Standards and the Mathematics Framework for California Public School (2013), we learned that the topics of multiplication and division make up a significant part of the math learning in third grade. This is the first year these topics are introduced and there is a lot of learning to accomplish during this time. Students must represent and solve multiplication and division problems with whole number products and quotients. They also learn important properties of multiplication: the commutative property of multiplication (a x b = b x a), the associative property of multiplication (a x (b x c) = (a x b) x c), and the distributive property (5 x 7 = (5 x 3) + (5 x 4)). They also learn that division and multiplication are inverse operations and that division can be thought of as an unknown factor problem (to solve a / b = ?, solve b x ? = a). As students learn the products of all pairs of one-digit numbers, they are expected to multiply and divide within 100 and to solve problems involving more than one operation.

As a multi-grade team, we were interested in how the third grade multiplication and division standards build on previous learnings and lay the foundation for future work. We used both the California Common Core State Standards and the Progressions for the Common Core State Standards in Mathematics (2011) as our foundation for this research. While the operations of multiplication and division are not formally introduced until third grade, we did find some significant foundation work in the primary grades. For example, skip counting is a practice and concept introduced as early as kindergarten, which lays the groundwork for multiplicative thinking. In kindergarten, students count to 100 by 1s and 10s and in first grade students focus on "making tens" and seeing "a ten" as a unit (the same way students must think of other quantities as units in multiplication, as in 7 x 6 is the same as 7 "sixes," where six becomes a sort of unit). In second grade, students expand their work with skip counting: they are expected to skip count by 2s, 5s, 10s, and 100s within 1000. In this work, students learn the sequence of the 2s times table and the 5s times table, which can aid them in third grade in beginning to grasp the concept of multiplication as well as to perform certain multiplication calculations. Additionally, second grade students begin to explore rectangular arrays and to use repeated addition to find the total objects in an array. This lays the foundation for understanding multiplication as repeated addition (and using that understanding to calculate). The use of arrays will also develop into using arrays as a model for multiplication and eventually towards using area as a model for multiplication.

In third grade, a significant part of building conceptual understanding is recognizing situations that can be described using multiplication and division. During this year students explore equal groups problems, problems involving arrays, and area problems. When the number of groups is known and the size of the groups (or number in each group) is known and the total is unknown, multiplication is used. For example, there are 4 boxes and 3 cookies in each box- how many cookies are there in total? (4 x 3 = ?). When the total is known and either the number of groups or the size of the groups is unknown, it is a division situation (or missing factor problem). For example, if 12 cookies are shared between 4 kids, how many cookies will each kid get? (12 / 4 = ?, or 4 x ? = 12). Or, if 12 cookies are shared between some students and each student gets 3 cookies, how many students can have cookies? (12 / 3 = ?, or ? x 3 = 12).

In an array situation, where the number of rows and the size of the rows are known, but the total is unknown, multiplication is used. For example, there are 6 rows of chairs and 3 chairs in each row, how many chairs are there altogether? (6 x 3 = ?). A division or missing factor problem would result when the total is known and either the number of rows or size of the rows is unknown.

Similarly, in an area model, when the length and the width of a space are known, you can multiply to find the area. For example, a room is 6 meters long and 3 meters wide, what is the area of the room? (6 x 3 = ?). Division or missing factor equations can be used to find either the length or width when the area and the other dimension are known. Area presents a new challenge in that it is a new type of measurement, which may be conceptually difficult for students to grasp. Area as a multiplication situation is unique in that the two factors are units of length and the product is a different type of unit- a unit of area.

Beyond third grade, students in fourth grade will go on to learn comparison or “times as many” situations as additional scenarios in with multiplication and division can be used. For example, Brigid has 5 cookies, John has 3 times as many cookies and Brigid- how many cookies does John have? In future grades students will also encounter multiplication as a way to solve combination problems (If there are 4 shirts and 3 pants, how many different possible outfits can be made?) and rate problems (If I read 3 books a night, how many books will I read in a week?). Beyond 3rd grade, students also learn to divide with remainders and to multiply and divide with numbers of more than one digit. In this multi-digit multiplication and division, fluency with single digit multiplication and division is key. Multiplication and division will also come into play in later years and the “10 shift” pattern is explored and students begin to work with both larger numbers (ten-thousands) and smaller numbers (ten-thousandths) and to apply their whole number fluency to these new calculations.

As a group we also explored different models for representing multiplication and division and the merits of each. We explored physical models, like counters arranged in each sets and tiles arranged in arrays, as well as representational models, like equal group drawings, array drawings, number bonds, tape diagrams, and number lines. We came to an agreement that, while we ultimately want to move students from concrete models to representational models and abstract representation, we want to do so in a mindful way. For example, a student who can skip count easily does not need to draw every item in an equal groups drawing, they can move on to writing a numeral. On the flip side, a student who is unclear about the meaning of division could probably benefit with more experience with concrete models, splitting counters into equal groups, rather than being pushed to copy a representational model that doesn't yet hold meaning for them.

Additionally, as students tackle more challenging concepts, it might make sense for them to step back to using more concrete models. As stated in the "Progressions" document, while "Grade 3 students can be encouraged to move as early as possible from equal grouping or array models that show all of the quantities to similar representations using diagrams that show relationships of numbers because diagrams are faster and less error-prone and support methods at Level 2 and Level 3"... at the same time… "Some demonstrations of methods or of properties may need to fall back to initially showing all quantities along with a diagram." (p.25). This information informed our decision to make counters available to the students in the early stages of this division unit, at the same time that we model and encourage representational and abstract models of division.

Finally, as a part of our research we consulted with Tad Watanabe, who gave us some powerful feedback, including introducing this question:

> *"[O]ne thing that's not quite clear is what is the nature of students' difficulty with quotative division. Your plan seems to suggest that quotative division is more abstract, but I'm not convinced it is - and I don't think there is any research that shows quotative division is more abstract and therefore difficult for students to understand…*
> *...In fact, you might be able to make a case that quotative division is easier for students than partitive division. So, I wonder if any difficulty students have with quotative division may have something to do with the order in which division is introduced. In this unit, as in Math International, you introduce division in partitive situations first. That's how division is defined. Then students have to extend the meaning of division to quotative situations.*
> *...Even in Japan, there are math educators (Professor Fujii of Tokyo Gakugei is one of them) who think division should be introduced in the quotative situation first, then partitive. They argue that it is easier for students to relate (or interpret) a partitive situation to quotative situation."* -Tad Watanabe

This question of the relative difficulty and the ideal order of instruction of the two types of division is really interesting to us. And as of yet, we don't have a clear answer. Our gut feeling was that quotative is more difficult, but we could stand to spend more time examining student work to get a better idea. We hope that participants in our research lesson can help us gain insight.

Tad also pointed out that the way we have structured our unit, we are defining division as partitive and then extending the meaning of the operation to include quotative division, which students have experience doing in other operations. For example, students first learn the "take away" situation of subtraction, and then extend the meaning to include comparative subtraction. There's a powerful connection for our school-wide learning here because this is exactly the topic we examined in our previous lesson study cycle in Spring of 2017, when we built an addition and subtraction unit for kindergarten based on different problem situations in these two operations. As our team examined the research around different addition and subtraction situations, we learned that different situations posed varying types and degrees of difficulty (comparison subtraction is much more confusing to model than take away subtraction, for example). Additionally, variations in word problem syntax, and whether or not that syntax lines up with the equation that represents the problem, present additional challenges. As our team continues to build our pedagogical content knowledge around the problem situations associated with each operation, we become better equipped to address the mathematical challenges word problems hold for our students. And in the planning of a Teaching Through Problem-Solving unit, we have the opportunity to introduce these problem situations in an intentional manner and sequence.

9. Unit Plan

Lesson	Date	TTP Problem	New Learning	Summary	Standards addressed
1	10/27	There are 15 stars in all and 3 circles. How many stars are in each circle?	We can use multiplication to find a missing factor.	Sometimes the number you're trying to figure out ISN'T the total.	3.OA.A.4 - Determine the unknown whole number in a multiplication or division equation relating three whole numbers. 3.OA.B.6 Understand division as an unknown-factor problem.
2	10/30	We have 12 cookies. If they are divided equally among 3 children, how many will each child get?	Understand the meaning of partitive division and how to write a division equation.	To share, we need to have equal groups. Division is an operation we can use when we want to share something equally.	3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g. interpret 56 / 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal

					shares of 8 objects each. 3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3	10/31	There are 20 strawberries. If 5 kids share these strawberries equally, how many will each kid get?	Understand how to find the answer to a partitive division problem	Division is a missing factor problem The answer to 20 ÷ 5 is the number that goes in the ☐ in the 5 x ☐= 20.	3.OA.A.3 (see above) 3.OA.B.6 (see above) 3.OA.B.7 Apply properties of operations as strategies to multiply or divide
5	11/1	RESEARCH LESSON There are 12 cookies. If we give 3 cookies to each kid, how many kids can share the cookies?	Understand the meaning of quotative division	If 12 cookies are divided so each kid gets 3 cookies, the cookies can be shared among 4 kids. This case can also be written with the following division math sentence: 12 ÷ 3 = 4.	3.OA.A.2 (see above) 3.OA.A.3 (see above)
6	11/2	There are 35 flowers. We are making bouquets with 7 flowers. How many bouquets can we make? There are 36 balls. How many	Students understand that quotitive division situations can be expressed with division math sentences, as well as the meaning of quotative division	We can use division to solve fair sharing problems where we don’t know how many groups.	3.OA.A.2 (see above) 3.OA.A.3 (see above)

		baskets do we need if we put 4 balls in each basket?			
7	11/3	There are 20 stickers. If we give 5 stickers to each person, how many people can we give stickers to?	Students understand how to find the answers to quotitive division problems	The Answer to 20/5 is the number that fits into the ___ in ☐x5 = 20	3.OA.B.5 (see above) 3.OA.A.3 (see above)
8	11/6	Victor and Jayla made word problems using the math sentence 6/2. Let's compare the word problems that they made.	Students understand that partitive and quotitive division can be consolidated as "division" and they can find answers to division calculations.	(two different 6/2) Finding 1 person's share and how many people something can be division among are expressed as division math sentences. (BOTH are division)	3.OA.A.3 (see above)
9	11/7	Some cookies in a box will be shared equally among 4 children. How many cookies does each child get? (when there are 8, 4, and no cookies)	Students are able to solve division problems in which the dividend is 0 or the dividend and divisor have the same numerical value (division with 0 and 1)	We can use division even when the divisor is 0 or when the divisor and the dividend are the same	3.OE.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using the properties of operations.
10	11/8	….multiplication problem….	Students must determine which operation to use based on the word problem situation.	In order to solve a word problem correctly, we need to understand if the problem is a multiplication situation or a division situation.	3.OA.A.1 Interpret products of whole numbers 3.OA.A.3 (see above)
11	11/9	….division problem with array...	Students must determine which operation to use based on the word	When the size of the rows in an array is unknown, we can use division	3.OA.A.2 (see above) 3.OA.A.3 (see above)

			problem situation.	to solve the problem.	
12	11/13	practice: various multiplication and division problems	Students practice determining the appropriate operation to match a word problem situation.	When we don't know the total, we can use multiplication to solve. When we know the total, but we don't know the size of the groups or the number of groups, we can use division.	3.OA.A.1 (see above) 3.OA.A.2 (see above) 3.OA.A.3 (see above)
13	11/14	multi-step problem	Students interpret and solve a multi-step word problem	Some word problems require two steps to find the answer.	3.OA.D.8 Solve two-step word problems using the four operations.
14	11/15	multi-step problem	Students apply their understanding of division and multiplication scenarios to solve a multi-step word problem.	In word problems with 2 steps, we have to read carefully to know what operations match the situations.	3.OA.D.8 (see above)
15	11/16	Power Builder			
16	11/17	Mastery			

10. Design of the Unit and Lesson

Students coming into this unit have a fairly solid sense of multiplication. Most are consistently understanding the idea of groups and number in each group. They tend to not confuse with addition. And some are definitely thinking abstractly about division (some have gotten to division in ST Math - a computer math program that students do during independent computer time). While they have started building fluency with 5, 2, 3, and 4, some are still having to count items and definitely don't have the larger multiplication facts memorized.

This unit is meant to introduce both partitive and quotative division, then give students multiple opportunities to practice these in combination with multiplication. Our theory of action this year is that by having a more solid understanding of multiplication, students will have an easier time mastering division.

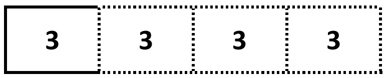
We have found that while the concepts may be clear, the language of word problems can still throw some of our students.

We pulled this lesson from the Japanese curriculum. Because the concept of division is still new, we are keeping the numbers small so that they can draw on manipulatives and drawings as needed. Using smaller numbers will also help the conversation focus more quickly on making sense of what the numbers are representing and how this problem compares to a similar one earlier in the week.

11. Research lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Stanford, I am loving how your reflections are helping me understand your thinking. I have a few I want to share today. "Let's think about what we did yesterday" -share 2 reflections Today we're going to try a new problem You know how I like to bake. You've had my brownies and my muffins. Well, Owen and I found a great recipe for cookies but it only makes 12 cookies! But now Owen has a problem. He wants to give 3 cookies to each friend. So here is the question:	redirection as needed	

Posing the Task Post on the board (have a student read) Owen has 12 cookies. If he gives 3 cookies to each friend, how many friends can share the cookies? "Think about it, think about what strategies you've learned so far and what you'd like to try, maybe a friend's strategy." Make sure I can tell what you find as the answer!	The problem will initially be written on the board. Students will also get a copy to glue into their notebooks. If a student notices "this is like the other division problem we did" … Did you hear what ___ said? Can you repeat it? (write on the board) Let's come back to it after we have tried the question.	Do students seem engaged and focused? Do students understand the task? Are they eager to solve the problem?
Anticipated student responses 2 minutes to glue, 7 minutes to solve Students may show a variety of strategies. These could include the following: **Direct modeling with manipulatives** Some students may use manipulatives to figure this out. (will be available on the tables) **Drawing groups:** Students may: Draw 12 and then group them Draw 3 and circle them, draw 3 and circle (repeated addition) until they have 12 **Another example of division with drawing or array…..** Some may use an array to organize their drawing, especially since we will be using the "pan" to show the problem. **tape diagram** --- --- --- --- --- -- -- -- **12** --- --- --- --- -- -- --	Teacher circulates and takes notes on who is using a particular strategy, lists out who they want to share (asking questions as ready to push farther to articulate) "What can you try based on our multiplication strategies?" "What can you try if you are stuck?" "Show me how you know." "Remember, we want to know what you're thinking. Show us what you're thinking." For those finishing quickly: "How else can you represent?" "How else can you show your thinking?" (looking back in journal and comparing to Monday, add to Tuesday)	What stage are the students in as they solve the problem? Concrete (using manipulatives), Representational (drawing pictures), or Abstract (creating number sentences)? Are students able to tackle the problem? Who uses manipulatives and how? (are they useful? not useful? what evidence do you have?) Who chooses which method? Are they labeling without prompting? What kinds of

 I have a few students (Tatiana for sure) who tend to draw tape diagrams. Interested to see if she uses for division problem. **Repeated Subtraction or Subtraction** or (labeling groups by skip counting) **missing factor problem** A few already seem comfortable with division (whether they understand conceptually is still TBD). They might do something like: ☐x 3 = 12 12 ÷ 3 = 4 (3+3+3+3 = 12) Some students might notice that this problem is 12 ÷ 3, just like a previous problem that's 12 ÷ 3. 3 circles 3 circles 3 circles 3 circles 4 groups of 3 cookies 4 people get 3 cookies each **Some might find incorrect answer - at least one of these:** - **12 = number of kids** - **12/4 = 3 kids** - 12/4 = 3 cookies - **12/3 = 4 cookies** - **12x3 = 36 Cookies** Students may be able to "solve" the problem and yet still not know the answer (for example, giving the total as the answer).These solutions point to the importance of really understanding the problem and what it is asking of us and	What does the ___ represent? Did you put a label? Would it help? During turn and talk, teacher circulates to hear who is able to articulate their ideas	misconceptions do you notice? What ways are students showing understanding?

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<td>the importance of labeling.

A few might start with an incorrect number of 36 counters or 15 counters
12-3 = 9
12+3 = 15
might show with different array (flipped)

It's time to share our thinking with our partner. What strategy did you use? Turn and tell your partner.</td>
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<td>Comparing and Discussing (Boardwork)
Transition to rug:

Misconception (only if many do - unlikely):
Possible misconception if strongly represented by 4 or more students
3x4=12 kids

Strategy 1 - direct modeling:
Direct modeling with manipulatives - Student puts 3 "cookies" in a group until all 12 cookies are gone.

note: if no students use manipulatives, may choose a student who draws 12 and circles groups of 3.

Strategy 2 representational:
Student draws 12 cookies and then proceeds to circle 3 cookies at a time until all are grouped.

Note: looking for someone representing the problem and labeling their work.
Strategy 2 might include:
● skip counting
● repeated addition
● starting with 12 and subtracting 3 at a time

Strategy 3 abstract:
12 ÷ 3 = ☐
☐x 3= 12 (if student doesn't write equation but solves thinking about it in terms of multiplication the teacher can</td>
<td>Potential teacher questions/prompts:

Students transition to rug:

Strategy 1 - Direct Modeling

Pretend like those are your blocks, show us what you did. Teacher labels the groups with the words the student uses to describe.

If confusing, ask: "What do others think?" or "What questions do you have?"

Responding to blocks:
How many blocks did you need? How did you know?

Ask certain kids to revoice (focus on the kids for whom this might be a struggle). "What did _____ just do?" Ask kids to respond to questions about misconceptions (why were there 12?)

*Note: Even if students aren't using the blocks to model, we will still use the cookie pictures to show the first solution (even</td>
<td>Are students able to articulate what the numbers represent?

Are they able to correctly identify the number of friends who can get cookies?</td>
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ask the class how they might represent the thinking with an equation)	though the kid might have just done a drawing). This could help make the solution more accessible to kids who are struggling.* **Strategy two: Representational:** Sharing drawing strategy "What does each number stand for?" "Let's make sure we understand all that ____ did. ____, what did ____ do?" "How is (example 2) similar to (example 1)? What is similar and what is different?" (turn to your partner) (what do we want to get out of those questions? Hoping students might see the how the representational maps onto the concrete. They may also notice that one starts with a total and one starts with a part. Students can understand that a given problem can be solved flexibly. Also that when you keep good track of what the parts of your solution represent, then you can find your correct answer. **Strategy 3:** "I'm so confused. What does this mean?? What does this represent?" What does this all mean? What are these three circles? What is this group? How can we write an equation? We hope someone will find connection between this problem and Monday's problem.	

	If they don't… find some sneaky way to bring it up, "I heard someone say this was the same problem from Monday." ;-) (If someone sees how similar to the prior problem) Explain what ___ said. "Do you agree or disagree? Why?"((Turn to your Partner) Follow with discussion Goal of getting to the difference between the two problems- and the importance of labeling to know what each number represents. If students notice that this is 12/3 like in previous problems, ask "Why 12/3 when we are not making 3 equal groups?" He wants us to help students understand the difference between "groups of 3" and "3 groups of _" What's different from how we've been dividing until now? What do you think of that? What does the 12 represent? What does the 4 represent?	
Summing up Summary: (may not be able to get to the comparison yet) This is a problem we can solve with division ~~If 12 cookies are divided so each kid gets 3, the cookies can be shared among 4 children.~~ ~~This case can also be written with the following division math sentence:~~ ~~12 ÷ 3 = 4~~ (not as good for ELLs)	What did we learn today? teacher jots down notes from what kid say, then make a summary from that. "What do others think?" Could refer back to poster of the unknowns in a multiplication and talk about unknown factors. "Did division work when we were trying to find the number of cookies? Did it work when we were trying to find the number of	Does the summary accurately represent the discussion and lesson?

Alternate summary: Today we learned a new division situation. We can use division both to find the # of groups or to find the # in each group. or: Today we learned a new division situation. We can use division to find the # of kids OR to find the # of cookies each kid gets. (but they might not need the specifics here)	people?" Focus on what most getting out of the lesson.	

12. Evaluation

- How are students representing their thinking? What models did students use to make sense of the problem and how did they represent them (e.g., blocks, drawings, equations, diagrams, arrays, etc.)?
- What strategies do students use to solve the problem?
- Did students understand that this was a different type of division problem? If so, what is their understanding of quotative vs. partitive division? How do we know? (labeling, conversation)
- Does quotative division seem to be more challenging for students to understand than partitive? Or does it seem easier? Or the same?
- What evidence do we see or not see of student-to-student interactions building student understanding?

13. Board Plan