Lesson Research Proposal for Fourth Grade Division with Remainders

John Muir Elementary School
Instructor: Sara Liebert
Lesson plan developed by: Sara Liebert, Rashida Carter, Joe Mannarino

1. **Title of the Lesson:** Interpreting the remainder in division

2. **Brief description of the lesson**

Research Theme:
Nurture students' mathematical agency and identity through the design of lessons that engage students in a Teaching Math Through Problem Solving (TTP) approach and the use of productive conversations.

Our Theory of Action: If teachers apply a Teaching Through Problem Solving approach (TTP), then students will deepen their conceptual understanding of mathematics. Increasing students' conceptual understanding will help to support procedural fluency in math. This will result in students beginning to see mathematics as accessible through effort and identify themselves as powerful math thinkers. Students are able to communicate their mathematical ideas, and revise and reflect on them in classroom discussion and journals.

3. **Goals of the Unit**

The purpose of this unit is to develop students' conceptual understanding of division, specifically their conceptual understanding of what the remainder of a division problem represents. We will build on what students learned in third grade, when representing multiplication and division problems and their fluency of multiplication and division within 100.

At the beginning of this unit we will play with a variety of division contextual situations, mathematizing them with multiplication and division equations, and keeping the numbers within 100 so that there isn't a great cognitive numerical load. We will focus on the use of fair share (partitive) division models first and then move into looking at quotative division models.

Then the unit will focus on developing students computational understanding of division algorithms using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Students will also be asked to begin to solve division problems in which the remainder must be interpreted and considered when providing an answer.

4. Goals of the Lesson:

Students will further their understanding of division with remainders by encountering a new situation - cases in which the quotient + 1 = the answer. Students will analyze the question in order to recognize the situation in the story as being different from situations with remainders they have experienced. Students build on work in previous multiplication and division lessons towards understanding the connection between their equations and the problem situation, including understanding what each number in an equation represents, and understanding the interpretation of the remainder in this new problem situation.

5. Relationship of the Unit to the Standards

Related prior learning standards	Learning standards for this unit	Related later learning standards
Grade 3: **3.OA.7 Multiply and divide within 100.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **3.OA Represent and solve problems involving multiplication and division.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8	**4.NBT.B.6 Use place value understanding and properties of operations to perform multi-digit arithmetic.** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.0A.3 Use the four operations with whole numbers to solve problems.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	**5.NBT.6 Perform operations with multi-digit whole numbers and with decimals to hundredths** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6. Background and Rationale

Division has always been a difficult topic for us to teach and for our students to understand. Students learn new ideas by connecting new learning to what they already know. They learn about multiplication before they are introduced to division, so it makes sense to build upon their existing experience and understanding of multiplication. While we want students to understand the inverse relationship between addition and subtraction, and to use that information when calculating mentally, we also want students to see this relationship with multiplication and division. Many students in this class are not fully comfortable with multiplication yet, which makes introducing division even more difficult.

When our team looked at the SFUSD Core Curriculum Unit 4.4, we found that the unit was broken down into three major lesson series. The first lesson series focuses on the relationship between multiplication and division, and extends division into numbers larger than 100. In the second lesson series students use the area model from multiplication and apply it to division as a missing side length. This leads first to an exploration of area models as tools to represent partial quotient, and then to a more abstract notation for partial quotient. The final lesson series in the unit focuses on the meaning of the remainder in varying division contexts, and also shows students how to represent the remainder as a fractional part of the divisor (a grade 5 CCSS standard). They learn that the remainder of a whole number division problem can be interpreted in a variety of ways that are dependent on context.

Due to our students' emerging understanding of multiplication, we wanted to begin our unit on 4th grade division by keeping our division problems within 100 then extend division into numbers larger than 100 in the final lesson series. In our lesson sequence, we will begin by introducing problems that involve the remainder. We want to do this to help students solidify their understanding of multiplication within 100 while building upon their previous knowledge of the relationship between multiplication and division. Although this is a review of third grade standards, the introduction of the interpretation of remainder is new learning for our students in fourth grade, which continues to move our students forward.

Our unit is designed so that students can continuously move back and forth between multiplication and division by making sense of problem situations that build on a known context. Throughout the unit, we want students to always be referring back to the situation, not to just check the answer but to also think about whether their solution makes sense within different scenarios; for an example of this see SFUSD Unit 4.4 LS3 Day 2. Furthermore, we want students to face problem situations that require sense-making, and that will push students to use models, equations, and strategies thoughtfully, and with enough detail, so that another person can follow their thinking and reasoning.

Our fourth and fifth grade team has made a shift in how we teach mathematics this year by using a Teaching Through Problem Solving (TTP) approach. This allows us to base our instruction on students' own ideas and to move the unit forward in response to their understandings and misconceptions. This teaching approach gives us the opportunity to spend more time in the beginning of the unit on models that are more concrete, and to introduce more representational or abstract models as students gain more understanding. In our division unit, we will begin with manipulatives and equal group drawings in order for our students develop robust understanding of division using concrete representations. Finally, once students begin to understand partitive and quotative division and the different situations, we will support students to move to tape diagrams and complex equations, for example. We will support students to be able to think of different ways they can interpret the remainder, regardless of the model they use.

Though our analysis we found that the SFUSD curriculum may be designed for the students who are more proficient with multiplication of large numbers than our students currently are. In the SFUSD unit 4.4 Division unit, students are moved much more quickly towards abstract and representational models using larger numbers, which are taught through direct instruction (i.e. *SFUSD unit 4.4 Lessons Series 1, Day 5*). The curriculum then presents problems that require students to solve them with specific models, regardless of context, as a way to practice that strategy (i.e. *SFUSD Unit 4.4, Lesson Series 2, Day 3*). Additionally, students calculate the quotient and remainder, using fractional representation (i.e. *SFUSD Unit 4.4 Lesson Series 3, Day 1*) which is not included in CCSS-M. We want students to be able to abstract the math from the context, and then revisit the situation to make sense of the calculations and the remainder. Because of this, at the start of our unit, we expect students to determine appropriate strategies based on the context, as we anticipate that their understanding of division will be deeply tied to what is contextually appropriate. We also want our students to uncover new strategies, models, and ideas, through inquiry, problem solving, collaboration and reflection.

Through curriculum analysis of 3 sources (SFUSD, Do the Math, and the Japanese Sansu curriculum), we noticed that, from the beginning of the division unit, Japanese Sansu and Do the Math curricula present division problems that require students to contend with the meaning of the remainder explicitly. The SFUSD curriculum 4.4 entry task ("Amusement Park Rides") exposes students to one problem with a remainder (40 divided by 6), but the meaning and interpretation of the remainder is addressed mainly in Lesson Series 3, where students are faced with problems that require them to analyze and interpret the quotient and remainder. Second, we noticed that the Sansu and Do the Math curriculum in particular kept the numbers within 100, which we felt would enable our students to master the new mathematical learning (remainders) while continuing to strengthen their emerging understanding of multiplication. If our students are learning the new concept of remainder, we do not want them to be struggling with the computation; instead, we want them to continue to build strong fluency of division within 100.

By teaching through problem solving (TTP), we've made the space for students to use strategies they are comfortable with. As we continue to see students' different strategies shared through partner talk, whole class dialogue, and board work, we push students to adopt new strategies as they are introduced by classmates and as they make sense to the students. Furthermore, we have connected a situation to each division problem that students are solving in order for them to think about how to interpret the remainder (will there be leftover, should they round the quotient up or round the quotient down, etc.). With every task, we ask students to consider which method they think is most appropriate for the situation. Depending on their strategy, we not only learn what students are understanding, but can get a picture of how much of our class is ready to move to the next strategy, and/or the next model. We believe this approach to teaching, along with the design of our unit, will build student's conceptual understanding of division as well as students math agency and identity.

The topic we have chosen for our public lesson focuses on students interpreting division with remainders by encountering a new quotative division situation - a case in which the quotient + 1 = the answer. At this point in the unit students will already have been exposed to both quotative and partitive division calculations involving remainders where the remainder is left over. Our problem is a modification of a problem that appears in the SFUSD curriculum: 62 fourth grade students are going on a field trip to the Exploratorium. They will travel by car. If each car holds 4 students, how many cars will be needed? (from SFUSD 4.4 Lesson Series 3 Day 3). We liked the locality of the problem, but felt that we needed to modify two aspects: the numbers, and the vehicle.

We wanted to change the numbers to ones that students would feel successful manipulating. Our school wide focus is on mathematical agency and identity. Through initial observations of the class in lessons in the unit leading up to the public lesson, we noticed that when the numbers were smaller, students felt more comfortable sharing and engaging in class wide conversation. When the numbers got larger, students tended not to challenge each other's ideas and to accept what was presented as truth, without really questioning the mathematical reasoning. We also wanted the focus of the lesson to be on the interpretation of the remainder, and so we felt that by having smaller numbers, we would create an opportunity for more of the class to engage with and think about the remainder, especially for students still relying on inefficient models (drawings, circles and stars, etc).

We wanted to change the vehicle because, when thinking about driving alone (3 cars with 5 students each, 1 remainder), we felt that our students would see that as a fun experience. But with boats, it's very difficult to row a boat by yourself. We felt that this would encourage more conversation about not only the number of boats needed (4), but how we would be seated in the boats (5, 5, 5 and 1, or, 4, 4, 4, and 4). We believe this situation will spark an interest from our students and provide moments of tension in the discussion of solutions.

Research and *Kyozaikenkyu*

As a fourth and fifth grade team, we were interested in how the third and fourth grade multiplication and division standards lay the groundwork for understanding multiplication and division of decimals and fractions in fifth grade. Furthermore, we had always felt that division was an area that was difficult for both ourselves as teachers and our students. We decided to use this as an opportunity to research and learn more about the teaching and learning of division. We used the *Common Core State Standards for Mathematics*, *Mathematics Framework for California Public School (2013)*, and *Elementary and Middle School Mathematics: Teaching Developmentally (9th Edition Van De Walle, Karp, and Bay-Williams)* as the foundation for our research.

As we studied the Common Core State Standards, we noticed that multiplication and division are a significant part of the math learning in fourth grade. In fourth grade, students apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations-in particular the distributive property-as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they also learn to select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; they understand and explain why the procedures work based on place value and properties of operations; and, they use them to solve problems. In the same way that students approach multiplication, students are tasked with using all of the strategies, models, and ways of thinking to find quotients involving multi-digit dividends. Finally, they learn to select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. *(from Common Core State Standards, Grade 4 Introduction).*

We wanted to understand what students are expected to know and understand from previous grades in order to better understand what students should be able to do and know in fourth grade. By looking at the way that multiplication and division are taught across grade levels, we hoped to be able to better think through where students are on the continuum of math learning, and to use that information to better scaffold the unit for our students. We decided to look at the *Mathematics Framework for California Public School (2013)* in order to try to understand the foundational work for multiplication and division in the primary grades.

Although the operations of multiplication and division of whole numbers are not formally introduced until third grade, we noticed significant foundational work for these concepts in the primary grades. Skip counting is a practice and concept introduced as early as kindergarten, which lays the groundwork for multiplicative thinking and repeated addition. In kindergarten, students count to 100 by 1s and 10s and in first grade students focus on "making tens" and seeing "a ten" as a unit (the same way students must think of other quantities as units in multiplication, as in 7 x 6 is the same as 7 "sixes," where six becomes a unit). In second grade, students expand their work with skip counting: they are expected to skip count by 2s, 5s, 10s, and 100s within 1000. In this work, students learn the sequence of the 2s times table and the 5s times table, which can aid them in third grade in beginning to grasp the concept of multiplication as well as to perform certain multiplication calculations. Additionally, second graders begin to use rectangular arrays to work with repeated addition, which lays the foundation for multiplication in grade 3. They use concrete objects, such as counters or tiles and pictorial representations, to add the rows or columns to arrive at the same answer (providing a way for students to think about the commutative property of multiplication) This also lays the foundation for understanding multiplication as repeated addition (and using that understanding to calculate). The use of arrays will also develop into using arrays as a model for multiplication and eventually towards using area as a model for multiplication and division.

In grade three, a critical area of instruction is to develop student understanding of the meanings of multiplication and division of whole numbers through activities and problems that involve equal-sized groups, arrays, and area models. Because multiplication and division are new concepts in grade three, students need opportunities to develop, discuss, and use efficient, accurate, and generalizable methods to compute. The goal is for students to use general written methods for multiplication and division that students can explain and understand (e.g., using visual models or place-value language). Students use multiplication and division within 100 to solve word problems (3.OA.3) in situations involving equal groups, arrays, and measurement quantities. Students are also expected to use various strategies to *fluently* multiply and divide within 100 (3.OA.7). The following are some general strategies that can be used to help students know from memory all products of two one-digit numbers.

Strategies for Learning Multiplication Facts **3.OA.7▲**

Patterns

- Multiplication by zeros and ones
- Doubles (twos facts), doubling twice (fours), doubling three times (eights)
- Tens facts (relating to place value, 5×10 is 5 tens, or 50)
- Fives facts (knowing the fives facts are half of the tens facts)

General Strategies

- Use skip-counting (counting groups of specific numbers and knowing how many groups have been counted). For example, students count by twos, keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the "count by" list and are able to start within it.
- Decompose into known facts (e.g., 6×7 is 6×6 plus one more group of 6).
- Use "turn-around facts" (based on the commutative property—for example, knowing that 2×7 is the same as 7×2 reduces the total number of facts to memorize).

Other Strategies

- Know square numbers (e.g., 6×6).
- Use arithmetic patterns to multiply. Nines facts include several patterns. For example, using the fact that $9 = 10 - 1$, students can use the tens multiplication facts to help solve a nines multiplication problem.
  9×4 = 9 fours = 10 fours – 1 four = 40 – 4 = 36
  Students may also see this as:
  4×9 = 4 nines = 4 tens – 4 ones = 40 – 4 = 36

Strategies for Learning Division Facts

- Turn the division problem into an unknown-factor problem. Students can state a division problem as an unknown-factor problem (e.g., $24 \surd 4 = ?$ becomes $4 \times ? = 24$). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplication and division involving a particular number can be helpful.
- Use related facts (e.g., $6 \times 4 = 24$; $24 \surd 6 = 4$; $24 \surd 4 = 6$; $4 \times 6 = 24$).

Adapted from ADE 2010.

From: *Mathematics Framework for California Public School (2013).*

In grade three the general written methods students use should be variations of the standard algorithms. Grade three also focuses on students solving equal groups and array multiplication problems. Multiplicative-compare problems are introduced in fourth grade. Students also begin to grapple with division in two different situations: partitive division and quotative division.

In grade 4 students begin to compare quantities multiplicatively for the first time. In multiplicative comparison problems, a factor multiplies a quantity to result in a different quantity (for example a is *x* times as much as b and can be represented as *a = b x n.* Additionally in grade 4 students solve various types of multiplication and division problems (see Types of Multiplication and Division Problems table) and need many experiences of these different types of contextual problems. In grade 4 tape diagrams can help students visualize and solve multiplication and division word problems. Tape diagrams are useful for helping students connect what is happening in the problem with an equation that represents the problem.

Table 4-3. Types of Multiplication and Division Problems (Grade Four)

	Unknown Product	Group Size Unknown[4]	Number of Groups Unknown[5]
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there altogether? **Measurement example** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally and packed inside 3 bags, then how many plums will be in each bag? **Measurement example** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed, with 6 plums to a bag, then how many bags are needed? **Measurement example** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example** What is the area of a rectangle that measures 3 centimeters by 6 centimeters?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example** A rectangle has an area of 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example** A rectangle has an area of 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example** A rubber band is 6 centimeters long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18, and that is three times as much as a blue hat costs. How much does a blue hat cost? **Measurement example** A rubber band is stretched to be 18 centimeters long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example** A rubber band was 6 centimeters long at first. Now it is stretched to be 18 centimeters long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ *and* $p \div a = ?$	$? \times b = p$ *and* $p \div b = ?$

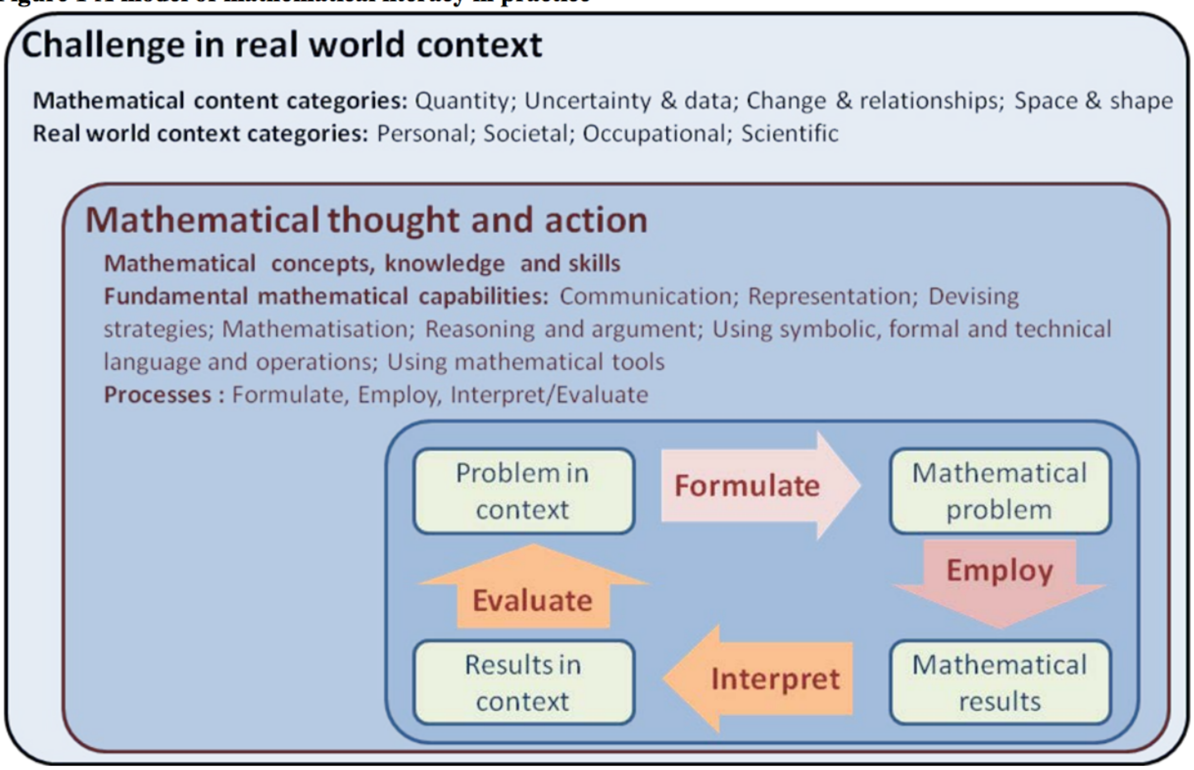
Source: NGA/CCSSO 2010d. A nearly identical version of this table appears in the glossary (table GL-5).

Additionally in fourth grade, students extend multiplication and division to numbers greater than 100. Students use models and drawing and connect those models to the equation in order to reason about the connection between the model and the equation. Students can use the area model to represent various different multiplication and division situations.

After reading Van De Walle, we became increasingly interested in the idea of presenting students in grade 4 with contextual problems involving division with remainders through a TTP approach to teaching and learning. According to Van de Walle, when teaching multiplication and division it is essential to use interesting contextual problems, rather than “more sterile story problems or naked numbers” (pg. 182). Van De Walle also emphasizes that the tendency in the US is to have students solve many problems in one class period with a focus on answer getting rather than understanding multiplication and division. Rather than do this, a better approach to teaching multiplication and division would be to shift the focus to sense making of the problem. One way to do this is to simply give students one word problem to solve that is both based on students experiences and builds on a known context. This shift allows students to use physical materials, drawing and equations to make sense of the problem and explain their reasoning with enough detail so that another person could follow along.

Throughout our research, we also explored the model of mathematical literacy in practice *(see figure 1: model of mathematical literacy in practice)*. We feel that our students are quite confident (showing math identity and agency) when given a mathematical problem in order to employ a result. However, we have noticed that when our students have to decontextualize and then recontextualize problems, many misconceptions and confusion arise.

Figure 1 A model of mathematical literacy in practice



from OECD - PISA 2015, Draft Framework, page 6 (https://www.oecd.org/pisa/pisaproducts/Draft%20PISA%202015%20Mathematics%20Framework%20.pdf)

As we developed our unit, we wanted to create problems in context that do not allow students to simply ignore the remainder. Van De Wall points out that the remainder in a division problem can have four different effects on the answer: (1) the remainder can be discarded leaving a smaller whole number answer (2) the remainder can force the answer to the next higher whole number (3) the answer can be rounded to the nearest whole number for an approximate answer and (4) the remainder can be interpreted as fraction. To effectively interpret remainders, students must attend to problem context and consider carefully what the remainder represents (Van de Walle, Karp, and Bay-Williams 2013). This means that when students are dealing with remainders, the context of the problem will determine the answer. The idea that the answer to a problem is not the solution to the equation is challenging for students. Battreal, Brewster, and Dixon (2016) provides examples of how the context of a problem can change the answer as students interpret remainders *(see table 1: Interpreting the remainder of a division problem)*.

TABLE 1

When dividing in real-world situations, the answer depends on the context of the situation and the particular question.

Interpreting the remainder of a division problem

Situation	Example	Answer
Drop the remainder	There are 58 students going on a field trip. Each van can hold 4 students. If all the students attend the field trip, how many of the vans will be full?	14 vans 14 vans have 4 students each in them. The last van has 2 students. Only 14 vans are full.
Add 1 to the quotient	There are 58 students going on a field trip. Each van can hold 4 students. If all the students attend the field trip, how many vans are needed to transport all the students?	15 vans To transport all the students, 15 vans are needed.
Remainder is the answer	There are 58 students going on a field trip. Each van can hold 4 students. If all the students attend the field trip and the vans are filled one at a time, how many students will be in the last van?	2 students Since 58 ÷ 4 is 14 remainder 2, then14 vans will be full (4 students in each van) with 2 leftover students. Since 2 students remain, they will need to be placed in the last van.
Remainder as a fraction	There are 58 cookies being shared equally with 4 students. How many cookies will each student get?	14 1/2 Each student will get 14 whole cookies and 1/2 of another cookie, because cookies can be evenly shared.

from 'When the Answer is the Question', Teaching Children Mathematics Vol. 23, No.1, pp. 30-37.

7. Unit Plan

Lesson & Date	TTP Problem	New Learning	Summary	Standards addressed
Entry Task 11/27	I have a 20 cm piece of tape and I want to make 2 cm pieces. How many pieces can I make? Is this is a division or multiplication problem?	Students need to make a decision and teacher can introduce tape diagram.		4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be

				interpreted CCSS.MATH.PRACTICE.MP4 Model with mathematics.
1 11/30	There are ___ cups of pudding and if we give 3 to each person, how many does each person get? (12, 15, 21) Can we use the multiplications facts of 3 to solve this problem? ***14÷3 = ?*** Leave lingering- students solve, but discussion will be lesson 2.	Think about how to perform calculations in cases of division resulting in a remainder that can be solved using multiplication facts. (Quotative)	No Summary, problem carries over to day 2.	4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them. CCSS.MATH.PRACTICE.MP4 Model with mathematics.
2 12/1	There are ___ cups of pudding and if we give 3 to each person, how many people can we serve? Students do practice problems.	Summarize how to perform calculations in cases of division resulting in a remainder that can be solved using a multiplication fact once. (Quotative)	We use the multiplication facts of 3 when finding the answer to 14÷3. There is no remainder=divisible and there is a remainder =indivisible.	4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. CCSS.MATH.PRACTICE.MP4 Model with mathematics.
3 12/4	There are 13 apples, if 4 are put in each bag, how many bags can you make? ***Analyze two students thinking: one of whom says 3 bags and 1 left over and one who says 2 bags and 5 left over** in order to determine who has more apples leftover to make another bag.	Understand the relationship between the remainder and the divisor. (Quotative)	The remainder of a division problem should be smaller than the divisor.	4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors CCSS.MATH.PRACTICE.MP4 Model with mathematics.
4 12/5	Leftovers Game			4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors
5 12/6	No Lesson (students on field trip)			

6 12/7	Four ways to solve 21 divided by 4 with Marilyn Burns I have 21 balloon to share with 4 people. How many balloons can each person get? I have 21 cookies to share with 4 people. How many cookies does each person get? I have $21.00 to share with 4 people how much money does each person get. Calculator: 21÷4=___	Think about how to perform calculations in cases of division resulting in a remainder that can be solved using multiplication facts. (Partitive)		4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted
7 12/8	We are going to cut a 60cm ribbon into 8cm pieces. How many 8cm pieces can we make and how many cm of ribbon will be left?	Understand how to check the answers of division calculations resulting in a remainder. (quotative)	If we multiply the divisor by the answer and add the remainder, it becomes the dividend.	4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors CCSS.MATH.PRACTICE.MP4 Model with mathematics.
8 12/11	There are 23 sheets of colored paper if we give 6 sheets to each person, how many people can we give the paper to.	Understand how to check the answers of division calculations resulting in a remainder. (quotative)	We can check to see if our division calculations makes sense using multiplication.	4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors CCSS.MATH.PRACTICE.MP6 Attend to precision.
9 12/12	There are 26 stickers. We are going to divide the stickers so that each child can get 6 stickers. How many children can get stickers?	Students understand how to use the context to represent the story problem with an equation.	It is important to look back at the story when we think about the equation that will help us solve the story problem.	CCSS.MATH.PRACTICE.MP4 Model with mathematics. CCSS.MATH.PRACTICE.MP6 Attend to precision.
10 12/13	There are 16 flower seeds. If we share them evenly among 3 people, how many seeds will each person get?	Students understand that indivisible cases of division can be applied in partitive	Using labels in my notebook helps me better understand the situation in the	4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including

		division as well.	problem.	problems in which remainders must be interpreted
11 Public Lesson 12/14	**SFUSD Unit 4.4 Adapted from Entry Task** ***There are 27 children. They are going to ride a roller coaster. Each car holds up to 6 passengers. How many cars do we need?***	Understand the remainder in problem situations. Students need to interpret the quotient in the context of the problem. (quotative)	We need to go back to the story of the problem to make sure the answer makes sense for the problem.	4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted CCSS.MATH.PRACTICE.MP4 Model with mathematics. CCSS.MATH.PRACTICE.MP6 Attend to precision. ***Focus on:*** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
9	We have 50 flowers. We are making bouquets that have 4 flowers each. How many bouquets with 4 flowers can we make?	Understand the remainder in problem situations. (Cases where the quotient=the answer).	Depending on the problem, we may or may not add 1 to the result of the calculation.	4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted CCSS.MATH.PRACTICE.MP4 Model with mathematics. CCSS.MATH.PRACTICE.MP6 Attend to precision.

8. Design of the Unit and Lesson

Students coming into this unit have recently finished a unit on multiplication. Beginning the multiplication unit with these students, we noticed that they did not have a strategies to fluently multiply within 100. They relied heavily on drawing circles and stars and repeated addition to find products within 100. Some students were still counting stars individually rather than seeing the equal groups as units. During the multiplication unit, we decided to provide students with multiple situations in which they used arrays to find products and split arrays to help them find products. These situations were connected to situations in which students were creating different sized candy boxes (lessons adapted from Marilyn Burns *Teaching Arithmetic: Introducing Multiplication)* and building arrays using grid paper. This led to solidifying students understanding of the commutative property of multiplication and the distributive property of multiplication as students learned to split arrays to create "easier multiplication problems". For example, students learned that you can split a 14 x 7 array into a 10 x 7 and 4 x 7 array, which results in computing 10 x 7 = 70 and 4 x 7 = 28, and then putting the arrays back together by adding the products 70 + 28 = 98.

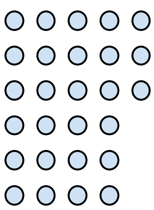
We designed this unit to build on students' recent understanding of multiplication and deepen their understanding of the connection between multiplication and division. We do this by beginning our unit by exploring division with remainders, providing an opportunity for students to extend their understanding of division--i.e., to learn that some numbers are not evenly divisible. Students' confusion about remainder provides an opportunity to deepen their understanding of the relationship between multiplication and division. In this unit, students will realize that unlike the array diagrams used when they learned about multiplication, when the numbers do not match up evenly, the leftover portion of the array is the remainder.

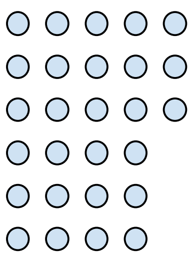
We start our unit focusing on division of numbers within 100-, and have tied these numbers to a story situation. Because when you introduce division you need to use multiplication, this allows students to access the problem through prior learning (recalling multiplication). We decided to introduce the remainder using a problem with smaller numbers, in order to allow students to grasp this important concept even as they continue to build their knowledge of basic multiplication facts that will be essential to multiplication and division of larger numbers. Our unit provides students extra time to access division through the multiplication, by emphasizing the connection between multiplication and division - not just reviewing. When students learn different division algorithms (area model, strategies based on place value, long division), they will only need to use division facts that are inverse of the basic multiplication facts - they just have to do it several times. If students have not learned division algorithms, trying to work with large dividends may put additional (and perhaps unnecessary) demands that may shift their attention away from the study of remainders.

We have designed this lesson to be taught using a Teaching Through Problem Solving (TTP) approach. We believe that this will deepen students' conceptual understanding of mathematics because students communicate and revise their mathematical ideas both in their math journals and with their classmates through partner talk and whole group discussions. Additionally we believe that this method, along with carefully designed lessons, will nurture students who see mathematics as accessible through effort and who identify themselves as powerful math thinkers. The design of the lesson is structured around *Figure 1 from OECD (Multiplication Action in Thought)* moving from abstract to more concrete. This is the reverse of how the board work is often structured. This choice was made because many of the students are relying too heavily on concrete representations and we want to push them towards the relationship of multiplication, division and the concrete representation.

9. Research lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Your reflections continue to help me understand your learning and thinking. Let's think about what we did yesterday. (Share out 3-4 students' reflections) How many of you have been on roller coasters before? How many of you like going on roller coasters? Today we have a new problem to think about. I want you to imagine that the class is going on a field trip to Great America. *There are 27 children. They are going to ride a roller coaster. Each car holds up to 6 passengers.* Introduce the word passenger and that you ride in cars for a roller coaster. (picture of a roller coaster) **Problem modified from* SFUSD 4.4 Entry Task	Student redirection as needed.	
Posing the Task **So here's the question:** *How many cars do we need?* (T write everything in red on board) Let's think about what math sentence represents this situation? Show me a thumbs up when you have an idea. (wait time) Turn and talk to a partner- what math sentence can we write to represent this situation? *As a whole group, students construct the math sentence: 27÷6 = _____* *total kids/kids per car= number of cars* *6 x _____ = 27* *kids per car x number of cars = total kids*	After posing the task, teacher circulates the room and takes notes on who is using a particular strategy, lists out who they will ask to present (asking questions as ready to push students farther to articulate their thinking) *What does the 27 mean in this problem?* *What does the 6 mean in this problem?* *What are we trying to find? What does the blank mean?* *During turn and talk, teacher circulates to listen to students ideas and to hear who is able to*	Observers should take note of how students are labeling their numbers in the notebook in relation to the problem. Do students seem engaged and focused? Do students understand the task? Are they eager to solve the problem?

	articulate their ideas. May ask students clarifying questions.	What prior knowledge do students use to try to solve the problem?
Anticipated student responses **Students may show a variety of strategies:** **Direct modeling with manipulatives** Some students may use manipulatives to figure this out. (will be available on the tables) Students also may not use manipulative but will model this in their notebooks.  Answer: 4R3 **Use repeated addition** 6+6+6+6=24 with 5 kids remaining *Answer:* 4 cars **Use their understanding of multiplication** and do 6 x 4 = 24 and 24 + 3 = 27, 6 x 5 =30 is too many. So, 5 is left and the answer is 4R3. *Answer:* 5 cars (Student has interpreted the remainder in the context of the problem) **Skip-Counting:** Students may use their understanding of skip counting and do 6, 12, 18, 24, 30. Answer: 4 or 5 cars because I counted by 6 4 or 5 times to determine how to get to 27.	*"What can you try based on our what you've done in your notebook?"* *"What can you try if you are stuck?"* *"Show me how you know."* *"Remember, we want to know what you're thinking. Make sure to show us what you're thinking."* ***For those who finish quickly:*** *"How else can you represent?"* *"How else can you show your thinking?"* *What does the ___ represent? Did you put a label? Would it help?*	What stage are the students in as they solve the problem? *Concrete (using manipulatives), Representational (drawing pictures), or Abstract (creating number sentences)?* Are students able to tackle the problem? What strategies are students writing down in their notebooks to show their thinking? How are students using their math notebooks as a resource?
Neriage: "Kneading" Discussion to Compare Solutions and Draw Out Key Mathematics **Board Work #1 Multiplication** Uses the multiplication sentence to answer the question:	**Potential teacher questions/prompts:** **Boardwork # 1:** Who used the multiplication sentence to figure this out that we	

6 x 4 = 24 6 x 5 = 30 Then the student realizes the 30 is too big, so 6 x 4 + 3 = 27. Therefore, 27÷6=4R3 **Board Work #2 Skip Counting** 6, 12, 18, 24 R3 6 x 4 + 3 = 27 Therefore, 27÷6=4R3 **Board Work #3 Concrete Model** Some students may use manipulatives to figure this out. (Counters will be available on the tables.) Students also may model this in their notebooks without using manipulatives.  Answer: 4R3 Math Sentence: 27÷6=4R3	used in the beginning? How did you know what number to use here? What does the 6 represent? What does the 4 represent? What does the 24 represent? What does the 3 represent? Do you understand this student's thinking? **Board Work #2 Skip Counting** What does each number in the skip counting represent? What division equation can you use to represent this? **Board Work #3** Why did you draw 27 dots? How did you break up the dots? Where does the 4 come from? What does each group represent? What does each dot represent? Where is the 27 in the diagram? **Guiding Question:** What does the 4R3 mean in our story?	
Summing up For Students: We need to go back to the story of the problem to make sure the answer makes sense.	4R3 is not the answer to the story of the problem so what should we do? We should look back at the story. What did we learn as a class?	

10. Board Plan