What's Hard About Fraction Number Sense?

Challenge in Understanding Fractions	Example of Student Difficulty or Understanding
A Fraction is a Number • A fraction represents an amount, not just pieces (such as 2 of 3 pieces of a pizza) or a situation (such as 2 of 3 shirts are red).	• When asked to put the fraction $^2/_3$ on a number line, a student said "you can't put it on a number line, because it's two pieces out of three pieces, it's not a number." Or " $^2/_3$ is not a number, it's two numbers." [*]
Fractions Can Be Greater than One • May be difficult for students who have a strong image of a fraction as a *piece of something*.	• "You can't have $^6/_5$ because there's only $^5/_5$ in a whole."
Fractions Can Be Partitioned • A whole can be divided into smaller and smaller equal parts. • The same fractional quantity can be represented by different fractions.	• Difficulty seeing how to divide a whole into *equal* parts. • Difficulty seeing that $^1/_2$ is equal to $^2/_4$, $^3/_6$, $^4/_8$, $^5/_{10}$ and so on.
What the Denominator Means • Different units (such as $^1/_3$ and $^1/_5$) are different sizes. • The more units a whole is partitioned into the smaller each one is. • $^1/_n$ fits exactly *n* times into the whole.	• Students add $^1/_3$ + $^1/_5$ and get $^2/_8$, without realizing they are adding two different things (thirds and fifths) sort of like adding apples and hammers. • Students may think "$^1/_5$ is bigger than $^1/_4$ because 5 is bigger than 4." • Difficulty seeing that $^1/_3$ fits in the whole 3 times, $^1/_4$ fits in the whole 4 times. Has trouble seeing that $^3/_3$, $^4/_4$ etc. equal 1.
Knowing What is the Whole • Constructing the whole when given a fractional part. • Keeping track of the whole.	• Difficulty making the whole when you give them a fractional part, e.g.: "This paper is $^2/_3$; show me the whole." • Sees that the magnitude of a fraction depends on the magnitude of the

	whole (e.g., half of a small cookie is not the same as half of a large cookie) • Confusion about whether the two drawings below together represent $^{3}/_{8}$ of a pie or $^{3}/_{16}$ of a pie. 
Fraction Size • Understands that fraction size is determined by the (multiplicative) relationship between numerator and denominator - not just by the numerator, not just by the denominator, and not by the *difference* between numerator and denominator. • Sees non-unit fraction as an accumulation of unit fractions. [A unit fraction has a numerator of 1; a non-unit fraction has a numerator other than 1.]	• May think $^{4}/_{9}$ is bigger than $^{3}/_{4}$ because 4 is bigger than 3 (comparing numerators), or $^{4}/_{9}$ is bigger than $^{3}/_{4}$ because 9 is bigger than 4 (comparing denominators), or $^{3}/_{5}$ is the same size as $^{5}/_{7}$ because the difference between the top and the bottom in both fractions is 2. • Sees that equivalent fractions have the same multiplicative relationship between numerator and denominator. In $^{2}/_{4}$, $^{4}/_{8}$, $^{3}/_{6}$, etc. denominator is two times numerator. • Sees $^{5}/_{8}$ is made up of five $^{1}/_{8}$'s or 5 times $^{1}/_{8}$, that $^{9}/_{8}$ is made up of 9 eighths or 9 times $^{1}/_{8}$, etc.

[*] Kerslake, D. (1986). *Fractions: Children's strategies and errors. A report of the strategies and errors in Concepts in Secondary Mathematics and Science Project*. Windsor, England: NFER-Nelson. Behr, M.J. & Post, T.R. (1992). Teaching rational numbers and decimal concepts. In T.R. Post (Ed.), *Teaching mathematics in grades K-8, research-based methods* (pp. 201-248). Boston: Allyn and Bacon.