SFUSD: Planning & Memorializing Template
MATH LESSON STUDY

Developed by Catherine Lewis and Shelley Friedkin (Mills College) and the San Francisco Unified School District's QTEA Team, 2017

I. RESEARCH LESSON OVERVIEW

School Name: Hillcrest Elementary
Lesson Study Team Members: Leila Christenson, Lisa Gaglioti, Susi Brennan, Karen Cortez, Amy Salfen
Title of Research Lesson: **Date of Research lesson: April 17, 2018** **Grade Level: Third** **Instructor's Name: Karen Cortez**
Essential elements for effective Lesson Study 1. A clear research purpose 2. Kyouzai kenkyuu (examination of curricula, standards, and progressions) 3. A written research proposal 4. A live research lesson and discussion 5. Knowledgeable others 6. Sharing of results Lesson Study Alliance; *Helping teachers work together to improve teaching & learning; http://www.LSAlliance.org*

II. RESEARCH CONCEPTION MAP

(copy directly from Designing Your Research Question and ToA)

MAP OF RESEARCH CONCEPTION

School Vision Statement or Educational Goals (including school math goals)

Students access prior knowledge to solve and understand new concepts in math.

Ideal Profile of Students

- Have multiple strategies to solve
- Persevere through new learning
- Use prior knowledge to attempt to solve a problem
- Explain their thinking
- Critique the reasoning of others
- Growth mindset

RESEARCH QUESTION

Is it necessary to put constraints on denominators when working with fractions in third grade?

Actual Profile of Students

- Feel confidence in math
- Tend to stick to the things they already do well
- Some not engaged in partner talk
- Few can critique others thinking
- Some are able to identify where they have made a mistake

THEORY OF ACTION

If we as teachers provide instruction around additional fractional parts outside of the CA Standards, then students will be able to compare other fractional units besides those listed in the 3rd grade standards, will have additional practice with parts of a whole and be able to extend that knowledge to their understanding of decimals when introduced in fourth grade.

METRICS TO MEASURE PROGRESS

Student work

Student journals and reflections

III. UNIT ANALYSIS

Unit Name: Fractions, Third grade

General Flow of the Unit:

This unit begins with students learning to identify unit fractions and also that there are multiple ways to write fractions. Students learn the components of a fraction. Next, students practice finding various unit fractions, then moves to fractions that represent a whole. Students will then learn to compare fractions with like denominators. The unit then moves to students learning about fractions that are equivalent to whole numbers. Students learn how to represent unit and non-unit fractions on a number line by partitioning the number line into equal parts.The unit then moves the students to learn about equivalent fractions and creating simple equivalent fractions. Finally, the unit compares and orders various fractions with either the same numerator or the same denominator.

Goals of the Unit ("Core Math"):

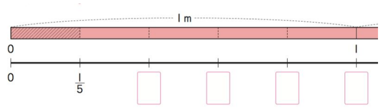
- Fraction of an area
- Unit Fractions
- Fractions on a number lines (interval 0 - 1)
    - Focus on labeling a number line (i.e. 0, 1 whole, 5/5)
- Fractions of a group (not technically a standard, want to give a basic overview)
- Equivalent fractions (building equivalency)
- Compare fractions (on a number line)

Lessons within the Unit:

Lesson	Learning Activities	Guiding Question	Summary	Standards
1	This is a 1 meter strip. You will have one part of this strip. What part of the strip did you get? Provide each student with a meter	This is the whole. How many of these smaller parts make the whole?	When 1 m is partitioned into 3 equal parts, each part is called one third of 1m. It is	CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a*

	strip and ● A half meter strip (½) ● A one quarter meter strip (¼) ● A one third meter strip (⅓)		written ⅓m and your read ita s one third of a meter. Numbers like ½, ⅓, and ¼, are called fractions	parts of size 1/*b***.**
2	How much of the whole is this strip? (⅔) ● A two thirds meter strip Ask question from Day 1 -- This is one whole strip. You will have one part of this strip. What part of the strip did you get?		When 1 strip is divided into three equal parts we call 2 of the parts two-thirds of the strip. It is written as ⅔ and is written as two thirds of the strip.	CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b***.**
3	Practice Day (not in RL plan) Geoboards (both square and circle side) ● show ½, ⅓, ¼, ⅙, ⅛ ● T delegates the whole (vary the size of the whole when S find different fractions) Show how to represent as a unit fraction Practice writing fractions on whiteboards (write “not a fraction” for fractions that aren’t fractions)	Geoboards 1st day - make it with grid paper and emphasize PROVING that the parts are equal		CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b***.**

	2 types of fractions
	• Part of group
	• Part of whole
	  ○ Circle
	  ○ Bar
	  ○ Number line			

4	Today's whole is 1 meter. How many meters are two, three, and four ⅕ m? (whole class launch) Then present Sansu Visual:  (student book 48) Modify the visual so that the number line doesn't have tick marks, but is just labeled with the 0 and 1. Because students will be identifying 5/5 we will also be discussing how this represents a whole number (1)	A) Let's express these fractions on a numberline. (Show both the meter model and number line) B)How many meters is five ⅕ meters? You can use your fractions strips if you need them.	We can use a number line to show fractions. We can write a whole number as a fraction. When the numerator and the denominator are the same it is one whole.	CCSS.MATH.CONTENT.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. CCSS.MATH.CONTENT.3.NF.A.3.C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6;* ***locate 4/4 and 1 at the same point of a number line diagram*.** CCSS.MATH.CONTENT.3.NF.A.2.A Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. CCSS.MATH.CONTENT.3.NF.A.1

				Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. CCSS.MATH.CONTENT.3.NF.A.2.B Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
5	Which is longer ⅘ meter or ⅗ meter?	How can you prove it?	4 ⅕ is ⅕ more than 3 ⅕ Dark line notation for number of parts on a number line	CCSS.MATH.CONTENT.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. CCSS.MATH.CONTENT.3.NF.A.2.B Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
6	Provide students with three 1 meter strips. How can we represent this with a fraction?	When we are given multiple wholes, how can we write that as	We can write multiple whole numbers as fractions. The	**CCSS.MATH.CONTENT.3.NF.A.3.C** **Express whole numbers as fractions, and recognize fractions**

		a fraction?	numerator is the total number of wholes and the denominator is 1. 3/1	**that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6;*** *locate 4/4 and 1 at the same point of a number line diagram*.
Math Talk	What are these different fractions on the number line represented by the tick marks A, B, C, D		Must expose to sixths, fourths, thirds, and eighths	CCSS.MATH.CONTENT.3.NF.A.2.B Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
Math Talk	How can we label the 10 cm tick marks on a meter fraction strip? (provide meter stick)		We can label parts of a meter with ten as the denominator	CCSS.MATH.CONTENT.3.NF.A.2.A Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
********* **7. Public Lesson** *********	Equivalent fractions		*There are fractions like ½ and 5/10 that are written differently but are equal in size.*	CCSS.MATH.CONTENT.3.NF.A.3.A Understand two fractions as equivalent (equal) if they are the same size, or the

*	Ms. Cortez ate 5/10 of a turkey sandwich. Ms. Leila at ½ of her turkey sandwich. Who ate more? 5/10, ½ WHOLE CLASS: All the ways you can represent ½			same point on a number line. CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
8	Ms. Cortez bought 2 pizzas. Saul ate ½ of one pizza. Ana ate ¾ of the other pizza. Who ate more? Comparing fractions (1) Unit fractions in different forms (circle, rectangle, etc…) (a) Size of the whole matters (e.g. ½ of a small pizza is not the same size as ½ of a large pizza); need the whole to be the same (2) Fractions with the same numerator (3) Fractions with different numerators and denominators (use ½ as the benchmark)	**EngageNY Module 5, L. 10** https://www.engageny.org/resource/grade-3-mathematics-module-5-topic-c-lesson-10 Play the game on p.113	We can compare fractions < less than > more than = equal to	CCSS.MATH.CONTENT.3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
9	Danny is giving away his Pokemon cards. He gives ⅙ to Nathalie. He gives 2/9 to Nayeli and gives ⅓ to		We can order the fractions using a model.	CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent

	Manuel. Order the fractions			fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. CCSS.MATH.CONTENT.3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
4th grade	Lesson 1 decimals p. 122 Sansu 3b. Student page 50	In third grade you learned how to write fractions on a number line. We have been studying so much about place value. How could we write these numbers that are smaller than 1? How can we write a fraction using place value?	Fractions with a denominator of 10 can be expressed as decimals.	

IV. BACKGROUND & RATIONALE ("Anticipate Student Responses")

*This section explains the flow of the unit and lessons in context of **your** students. What do students currently understand about this topic? What do they struggle with (based on past experience)? How will the lessons of the unit, including the research lesson, help them develop the understanding outlined in the unit goals?*
This is the 7th lesson involving fractions in 3rd grade. Starting in 1st and 2nd grade, they learn to partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. In 3rd grade they understand fractions as a number, fraction of an area, use fractions on a number (interval 0-1), find simple equivalent fractions, and learn to compare fractions using a numberline or visual model. All fractions are proper and the standards include denominators 1, 2, 3, 4, 6, and 8. At this point of the unit, students have seen various fractions, but have not been exposed to equivalencies. Students typically struggle to find equivalencies and generate mathematically sound models that are helpful when comparing. For example, students struggle to create comparison models using the same size wholes. In this lesson the students will be asked to compare two different fractions that are equal to the same size. The rationale for using a number line is that students are very comfortable with addition, subtraction, and elapsed time on a number line and have practiced representing fractions on a numberline prior to this lesson. By asking students to model equivalencies, students will show understanding of fractions as numbers. Students will be able to demonstrate their understanding of fractions as a number, using a number line.

V. RESEARCH & KYOUZAIKENKYUU ("Anticipate Student Responses")

This section describes what you learned from examining the SFUSD curriculum materials, additional curricula or reference sources, and Common Core State Standards and Progressions. What did you learn, and what are the implications for your research lesson?

We recommend exploring the following in relation to your lesson/unit content:

1) California Common Core State Standards for Math
2) California Mathematics Framework
3) Progressions Documents for CCSS-M
4) Math Misconceptions (book)
5) Elementary and Middle School Mathematics: Teaching Developmentally (book)

6) Principals to Action: Ensuring Mathematical Success for all (NCTM book)
7) 5 Practices for Orchestrating Productive Mathematical Discussions (NCTM book)
8) Engage New York- https://www.engageny.org/sites/default/files/resource/attachments/math-g3-m5-full-module.pdf

Relationship of the Unit to the Common Core State Standards (CCSS)

Key Related Prior Learning ("Prior Supporting Mathematics")	Learning in This Unit ("Current Essential Mathematics")	Future Learning ("Future Mathematics")
2nd grade Fraction Standards: **Measurement and Data** Relate addition and subtraction to length. CCSS.MATH.CONTENT.2.MD.B.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. **Geometry** Reason with shapes and their attributes. CCSS.MATH.CONTENT.2.G.A.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half	**3rd grade Fraction Standards:** Develop understanding of fractions as numbers. CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. CCSS.MATH.CONTENT.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. CCSS.MATH.CONTENT.3.NF.A.2.A Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.	**4th grade Fraction Standards:** Extend understanding of fraction equivalence and ordering. CCSS.MATH.CONTENT.4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. CCSS.MATH.CONTENT.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the

of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. CCSS.MATH.CONTENT.3.NF.A.2.B Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. CCSS.MATH.CONTENT.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. CCSS.MATH.CONTENT.3.NF.A.3.A Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. CCSS.MATH.CONTENT.3.NF.A.3.C	two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. Build fractions from unit fractions. CCSS.MATH.CONTENT.4.NF.B.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. CCSS.MATH.CONTENT.4.NF.B.3.A Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. CCSS.MATH.CONTENT.4.NF.B.3.B Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*. CCSS.MATH.CONTENT.4.NF.B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing

	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. CCSS.MATH.CONTENT.3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. CCSS.MATH.CONTENT.4.NF.B.3.D Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. CCSS.MATH.CONTENT.4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. CCSS.MATH.CONTENT.4.NF.B.4.A Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*. CCSS.MATH.CONTENT.4.NF.B.4.B Understand a multiple of a/b as a

		multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* CCSS.MATH.CONTENT.4.NF.B.4.C Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* Understand decimal notation for fractions, and compare decimal fractions. CCSS.MATH.CONTENT.4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2*For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100*.

		CCSS.MATH.CONTENT.4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. CCSS.MATH.CONTENT.4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. 1 Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. 2 Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. **Measurement and Data** Solve problems involving measurement and conversion of measurements.

		CCSS.MATH.CONTENT.4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

Other learnings from studying curriculum materials, additional curricula, mathematical progressions and frameworks, and other resources:
Van de Walle, Sansu Math grade 3 and 4, Mills Fraction unit for grade 3 and 4, Teaching Through Problem-Solving website (http://www.lessonresearch.net), Engage NY

VI. GOALS FOR THE RESEARCH LESSON

We recommend utilizing the SFUSD Dimensions for Teaching and Learning to support development of your goals.

Brief description of the lesson selected:
The lesson selected is the first time the students will be working with simple equivalent fractions. At this point of the unit, students have seen various unit fractions and some non-unit fractions, but have not been exposed to equivalencies. In this lesson the students will be asked to compare two different fractions that are equal to the same size.

Lesson goals ("Core Math"):
As a result of this lesson, students will understand that even though two fractions do not have the same numerators and denominators, they can be equal. Students will also understand that you have to make sure to compare the same size model when working with equivalencies.

Summary: *There are fractions like ½ and 5/10 that are written differently but are equal in size.*

Goals related to our research question/theory of action:
How will this lesson facilitate student development related to our team or school Research Question and Theory of Action?
This lesson will facilitate student development related to our Research Question, whether students would be able to compare fractions using tenths in third grade. In the unit, students will be exposed to fractions with denominators 2, 3, 4, 6, and 8, as listed in the CCSS. They will be able to compare these fraction and represent them in various models. The goal is that students develop flexible thinking and not be limited by these denominators and begin seeing denominators such as 10 so that they can begin to understand that a number can also be broken up into more than 8 equal parts and that there are also fractions for more than just 2,3, 4, 6, and 8. Another goal is that exposing them to the denominator of 10 will help to give them background knowledge of tenths so that when they begin working with decimals in fourth grade, they will understand that a number can be broken up into tenths.

In the math talk prior to the public lesson, the students will be labeling 10 cm tick marks on a meter stick with fractions. They will then use this idea of dividing something into ten equal parts in the public lesson. The public lesson will facilitate the development of comparing tenths and halves because they have to understand that they are comparing two items of the same size that are divided into different fractional parts. The lesson will show that the limitations of the denominators 2, 3, 4, 6, and 8 do not have to be placed on students when comparing fractional parts. If the students are taught to divide a whole into fractional parts using these denominators and can identify these fractions, then they will be able to compare using fractions such as tenths.

Goals related to the pedagogical practices we are investigating (if different from above), such as whole-class discussion and mathematics journals:
What pedagogical practices are you investigating, and how? How will you assess progress or success? For example, if you are trying to build journal use, you might attach journal copies from your three selected students over the unit, and explain how you assess their quality.

Student progress and success will be assessed using their students journals, their explanations of their strategies, and the discussion during the debrief where they justify their thinking. Students are very familiar with journals and have been using the journals for two years. They will copy down the problem while it is introduced and work independently at their desks. Student work will be monitored as they are working. A progression of student work will be examined on the board. We will be able to observe their strategies, and then listen to and question them about their thinking. Students will also be able to question each others strategies. In these discussions, we will be able to assess if the students are understanding the idea of comparing fractions using tenths.

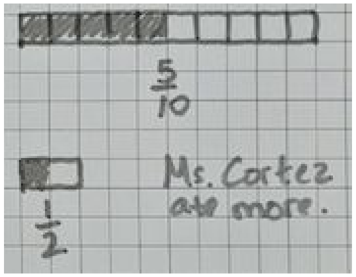
VII. LESSON DESIGN

Three Levels of Teaching	***Problem Solving*** ***(Standards and Focal Points, NCTM)***
Japanese mathematics educators and teachers identify three levels of expertise of mathematics teaching: **Level 1:** The teacher **can tell** students the important basic ideas of mathematics such as facts, concepts, and procedures. **Level 2:** The teacher **can explain** the meanings and reasons of the important basic ideas of mathematics in order for students to understand them. **Level 3:** The teacher **can provide students with opportunities** to understand these basic ideas, and support their learning so that the students become independent learners. *(Sugiyama, Y. 2008, Trans. Takahashi, A., 2011a)* *Lesson Study Alliance: http://www.LSAlliance.org* *Helping teachers work together to improve teaching & learning*	Problem solving means engaging in a task for which the solution is not known in advance. Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts. Students need to develop a range of strategies for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. *Lesson Study Alliance: http://www.LSAlliance.org* *Helping teachers work together to improve teaching & learning*
Explain why you have designed the lesson flow as shown in the next section. For example, what is your rationale for the selection and sequence of student responses? Incorporate what you learned from solving and discussing the mathematical task yourselves and anticipating student responses. Refer to the SFUSD Dimensions for Teaching and Learning which reference 3 dimensions critical to	

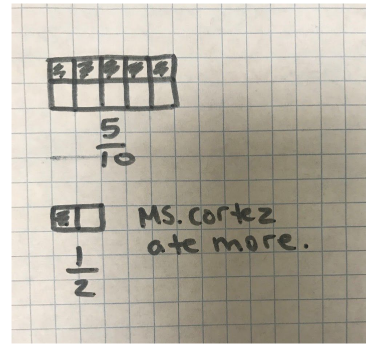
consider while planning: "Agency, Authority, and Identity"; "Access to the Content"; "Uses of Assessment". See also page 10 of the *SFUSD Strategic Plan (TLTL)* *for further explanation of the Dimensions.*
Rationale for *Agency, Authority, and Identity*: Student voice will drive the lesson. Every student will have time to struggle through a non-routine word problem. Student work will be used to reach the lesson objective and students will generate the lesson summary.
Rationale for *Access to the Content:* *What choices did you make in your lesson design to ensure access to the content for all students? Why?* The problem was chosen because it is something the students can relate to and access since they know the two teachers and it is a real-life situation which the students are familiar and which they are very engaged in solving for the teachers. Understanding equivalent fractions is new learning for the students, thus will be cognitively challenging. We have chosen numbers, such as the benchmark fraction ½, because the students have worked with ½ and can identify that. The fraction 5/10 will be more difficult to represent, but will facilitate an opportunity for the students to construct new ideas of equivalencies and around the idea that we need to ensure we are comparing the same size whole. This lesson design provides access to content for all students because the three read protocol is used giving all students a chance to understand the language, the quantities and the problem posed; the students are asked to use their own strategies to solve the problem and explain their thinking and are not instructed how to complete it; it gives them a chance to grapple with new learning and develop new ideas for why the fractions are equivalent; and finally because the students have to discuss the rationale for their answers and explain their thinking. Students will be allowed to use whatever strategy they need and will also be allowed to use math tools if necessary. The teacher will check in with certain students to ensure all students are accessing the content.
Rationale for *Uses of Assessment*: *What choices did you make in your lesson design about the use of formative and summative assessment? Why?* We will use a formative assessment for this lesson based on the work provided in the student journals. Throughout the lesson the teacher will monitor student understanding both verbally and written. The teacher can use the information gathered from the journals to guide students and assess their understanding of the lesson and to address misconceptions as they arise. In addition, there are many opportunities throughout the lesson where the students are asked to share with a partner or share their thinking whole class. These discussions are other opportunities for the teacher to monitor understanding and guide the questioning or prompting. The various assessments throughout the lesson allow the teacher assess student learning, guide the lesson and adjust the remainder of the unit if changes are needed.

VIII. FLOW OF THE RESEARCH LESSON

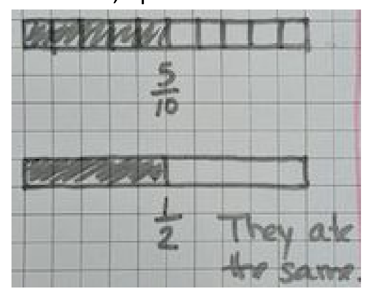
Components of the TTP Lesson Structure	Learning Activities	Anticipate Student Response
LAUNCH: Introduction (Hook) **LAUNCH: Posing the Task** **EXPLORE: Independent Work & Anticipated Student Responses** **("Monitor, Select, Sequence")**	Ms. Cortez and Ms. Leila always forget our lunches. We went to the Hillcrest Market to get sandwiches. We always get the same sandwiches. The lady who makes the sandwiches makes sure we get the exact same size. I always get interrupted while I'm eating and so I told the lady to cut it into small pieces. Then the lady asked Ms. Leila if she wanted hers in small pieces too. But Ms Leila doesn't get interrupted because she's always in my room and her kids don't know where she's at. So she said she just wanted big pieces. I ate 5 small pieces of my sandwich and Ms. Leila ate 1 big piece of her sandwich. So who ate more?	
	T: Turn and Talk T call on a student that said "We don't know because we don't know about many pieces each sandwich had altogether."	S: You ate more because you had 5 S: Ms. Leila ate more because she ate a really big piece S: We don't know because we don't know about many pieces each sandwich had altogether.
	T:Ok we need to know how many pieces. I actually brought a picture that can help us figure this out. This is how my sandwich was cut and this is how Ms. Leila's sandwich is cut. → PUT UP VISUAL (label Ms. Cortez and Ms. Leila)	
	T: So I ate 5 pieces of my sandwich and Ms. Leila ate one of her pieces. How much of my sandwich did I eat and how much of her sandwich did Ms. Leila? T: Turn and Talk Teacher writes the problem on the board, depending on what student response is.	S: 5/10 and ½ S: 5 pieces and 1 piece

	Ms. Cortez ate 5/10 of a turkey sandwich. Ms. Leila ate 1/2 of her turkey sandwich. Who ate more? OR Ms. Cortez ate 5 pieces of a turkey sandwich. Ms. Leila ate 1 piece of her turkey sandwich. Who ate more? Guiding question (If students are able to identify the fractions): Compare 5/10 and ½	Students are copying the problem into their notebooks as they read the problem
EXPLORE: Partner/ Group Work?	Students begin working independently. Teacher walks around observing student work. (T will check in with table groups. If SS start finishing up early they can work on their packets. T will remind them of this independently.) ** If majority of students only compare 5 and 1 then do a mid-workshop interruption*** T: I noticed that you put 5 is greater than 1. Let's read the problem one more time. T will also refer students back to the visual. T: Turn and tell your partner what do you think about this? T: Oh we are actually comparing 5/10 and ½ ⟶ (update problem on board to show 5/10 and ½)	Students work independently at their desks. S: Ohh they are 5 small pieces of 10. So it has to be 5/10. S: Oh it's not just 1, it's ½. Students continue working on the problem. Board Work: Student A, option 1 

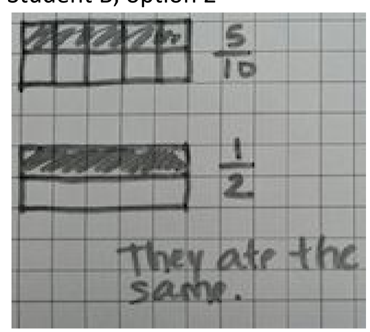
Student A, option 2



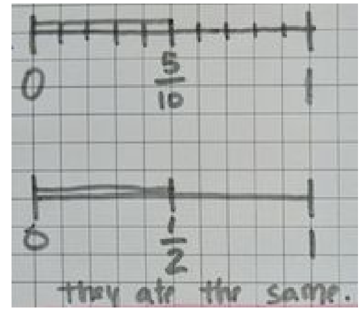
Student B, option 1



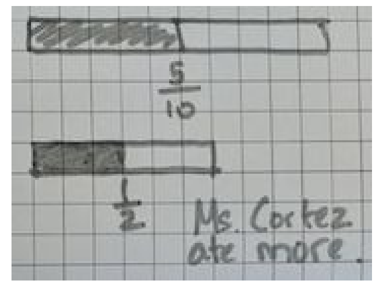
Student B, option 2

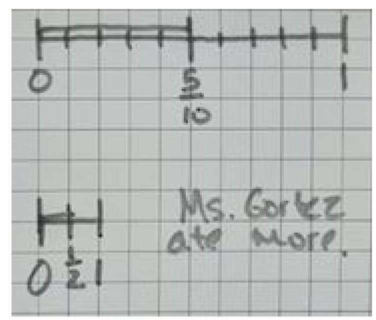


Student C



Other possible responses





After solving the problem they will record their answer in their notebooks.

		Recording strategies: strategy, labeled parts and answer sentence.
CONNECT: Neriage: "Kneading" Discussion (to Compare Solutions and Draw Out Key Mathematics)	T calls students back to the rug with notebooks. T: I saw so many different strategies and I'm excited to figure out who ate more! Teacher calls up student A T: Can you explain what you did? (T draws on magnetic grid paper) T: So who ate more? T: I noticed that some of you agreed and some of you disagreed. This will be an interesting conversation. T: Turn and tell your partner what you think about this visual model.	Student A1: I drew a fraction strip I used 2 squares. I shaded one of them in I drew another fraction strip used 10 squares and shaded 5 in . Then I labeled, the first one ½ Ms Leila. Then I labeled the second one 5/10 Ms. Cortez. Student A2: I drew a visual model just like the first one on the board with five boxes on top and 5 on bottom. I shaded the top five in. Then, next to that, I drew a box with two squares and shaded in the first one. Then I labeled the first one Ms. Cortez and the second one Ms. Leila. Student A: Ms. Cortez ate more. [Other students will use the hand signals for agree and disagree as the Student A is sharing their idea] S:I agree, I drew the same thing, it shows ½ and 5/10 S: I disagree because they are not the same size whole

		S: I wonder why student A did not remember they are the same size sandwich
	T: Super interesting, I heard that some students agree it is just like their model. Some had questions about the model and weren’t sure.	
	Teacher calls up student B T: Can you explain what you did?	Student B: I drew a fraction strip that was 10 squares long and shaded in 5 of them. Then I drew a fraction strip that was the same size as the 10 squares. Then I shaded in half. Student B2: I drew a visual model that was 10 squares with 5 on top and 5 on bottom. I shaded in the top 5 squares. I labeled it Ms. Cortez and I labeled it 5/10s. Then I drew a visual model the same size as Ms. Cortez’, and divided it in half. Then I shaded in ½, the top piece. I shaded in five squares on top. I labeled it Ms. Leila.
	T: How did you know to shade in five for 1/2?	Student B: Half of 10 is 5.
	T: So, who ate more?	Student B: They ate the same.
	T: How do you know we ate the same amount?	Student B: They both are five squares. Or you both have the same amount shaded in.

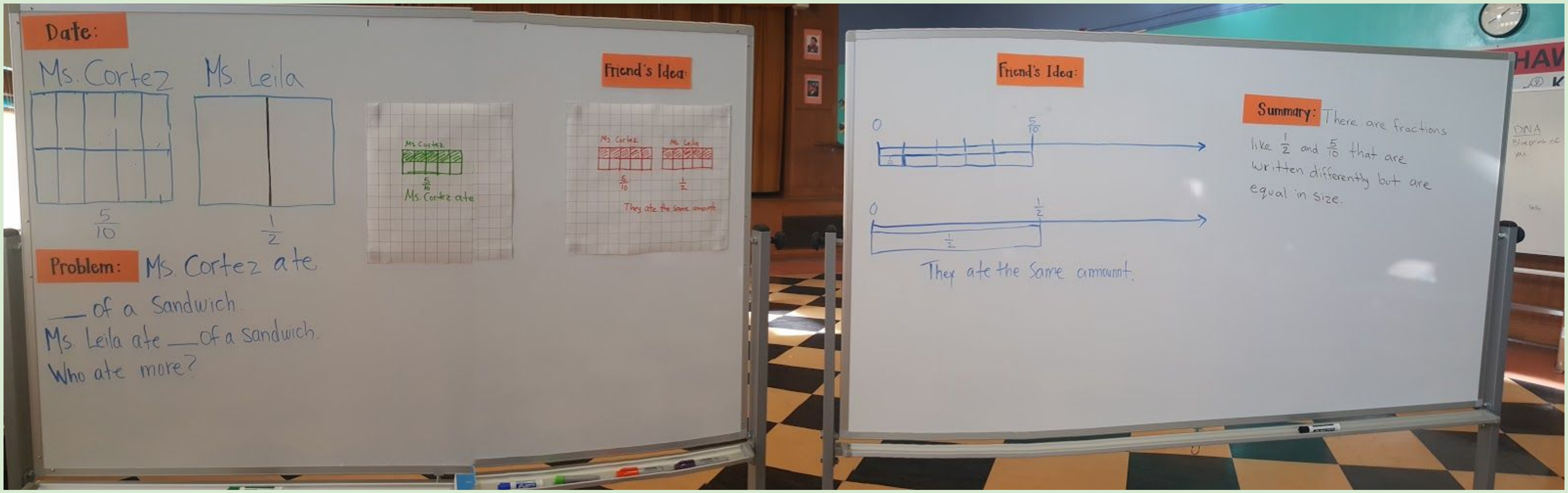
	T: Turn and tell your partner what you think about this model? T: Which of these two represent the problem? T: This doesn't show the same size sandwiches so student A's model doesn't match our problem. T: When looking through your work, some of you did this model	S: I agree, I drew the same thing. S: I disagree, Student B shaded in 5 squares for Ms. Leila and it's one. S: Ohhh, I did it wrong the sandwiches need to be the same size. S: I agree because half of 10 is 5 S: The sandwiches are the same size so the visual models have to be the same size. S: Student B because we know the lady made two sandwiches that are the same size S: Student A doesn't match the problem because they are not the same size S: To compare these two sandwiches they have to be the same size because that what the problem says

	(B) and some of you did something different but came up with the same answer. [Call up student C] T: Can you explain what you did? T: Who ate more? T: How do you we ate the same? T: Turn and tell your partner: What is the same and what is different about these two representations? (T & T) T: Calls on student to say "they both have the same size sandwiches and have the same answer"	Student C: I took out my fraction kit and I took out my tenths. I made a numberline and used the 1/10 to split the number line into ten pieces. I counted 1/10, 2/10, 3/20, 4/10 and 5/10. I shaded in 5/10 and labeled it Ms. Cortez. Then I took out my ½ because Ms. Leila ate ½. I made a numberline and used it split it into two pieces. I shaded in ½ and labeled it Ms. Leila. S: They ate the same amount. S: I looked at 5/10 on the numberline and it lined up with ½ on the other number line, so they were equal and I know they ate the same amount. S: One used a fraction model and one used a number line. S: They both had the same answer. S: They both have the same size sandwiches/wholes.

	Turn and tell your partner if you agree that this is the correct answer. T: I notice we all agree with this answer, but Ms. Leila ate ½ and I ate 5/10. What does that tell us about the fractions 1/2 and 5/10? T: So this tells us that 1/2 and 5/10 are the same amount! T: We ate a different number of pieces, but we actually ate the same amount. How come? (T&T) T: We found 5/10 is the same as ½. Some fractions may be written differently but represent the same amount. T: Today we were able to solve using both the visual model and the number line, I want you all to write both of these models in your friend's idea. (Don't forget to use your blue pen) Don't forget to update your work	S: Yes this is the correct answer. S: That they are equal amounts. S: That they are the same. S: Because Ms. Cortez ate 5 small pieces and Ms. Leila ate 1 big piece. S: copy model B and model C in Friend's Idea.
SUMMARIZE: Summing up	T: What did we learn about 5/10 and ½? Teacher writes summary on board: *There are fractions like 5/10 and 1/2 that are written differently but are the same amount.* T: Let's all read the summary together.	S: We learned that they are the same amount. Students read it together and copy the summary into their notebooks.
SUMMARIZE: Reflection		Students go back to their desks and write their reflection.

IX. BOARD PLAN

A board diagram shows how work on the board will be organized to support the lesson flow above. Conducting a mock-up lesson (with adults) and creating a board diagram can provide a chance for your team to think through the drama of the lesson–what students will say and do (monitor) and how and why students' thinking will change over the lesson (select and sequence) - and plan the board accordingly. For a board plan template and examples, see http://lessonresearch.net/ttp/boardorganization.html



X. EVIDENCE/DATA TO COLLECT OR OBSERVE

What data will help you determine what students learned and are able to do as a result of your lesson?

What data will help you assess student progress toward the lesson goals and your research question?
What data will help you assess student experience (interest, safety, agency, confidence, etc.) throughout the lesson?
(Consider student work, student responses and conversation, patterns of participation, student journals, student engagement, etc.)

We will observe and collect the following data to assess student understanding:

- Student work in math journals
- Student turn-and-talks
- Student reflection in math notebooks
- Students engagement
- Student discussion and summary
- Student confidence when solving the problem and discussing their strategy

XI. NOTES OR SCRIPTING FROM THE PUBLIC RESEARCH LESSON

Participants:Lauren G. Sara L. Tad W. Amy S. Leila C, Karen C. Lisa G. Susi B. Phil Daro. Nora H

Note-taker:Nobie C

Pre-Lesson Discussion Notes:

Scripting

- What is your **research question** and **theory of action**? Why did your team choose this focus? Why is this focus important to student learning and/or student experience at this site? What is the **context and rationale** for this research lesson? Why is it important and relevant for students? How is it connected to the **research question** and **theory of action**?
- Tad did a great job of intro… Is it necessary to put constraints on the denominator when working with fractions in 3rd grade. A lot of this has to do with a 4th grade problem we had in the fall with launching decimals. One was measurement would be a great way to launch decimals, but measurement happens in the spring, so we used money. We are wondering about how we can support the development of understanding of decimals with fractions. (Cut and paste ToA here). In our 4th grade public lesson, we struggled with finding a real life context for teaching decimals in 4th grade. We chose money. It didn't work out as we planned. Students seemed to be confused about the relationship and unclear about the meaning of decimals. From the confusion, we were able to wrestle with the questions of alignment and support between the concept of fractions and

decimals in 3rd and 4th grade. In 3rd grade, we also have some measurement requirements. We explore volume. We decided that decimals would be a better fit for our goals and more feasible. This lesson is backwards planned from some of the 4th grade goals.

- What are your specific **student learning objectives** and **intended student learning outcomes**? What are you hoping or expecting students to **do or say**?
- We hope that students will see that two fractions can be equal even with different numerators/denominators. We hope they see that the model has to be the same size. We hope that they use the number line. We are curious to see how they interact with 10ths - if it will be something they can manage. The unit has been built around the idea that fractions are numbers. Will students use fraction strips? Will they use number lines? Will the use earlier models ? Especially we are putting up a visual model for the context of the problem, will students rely on the model or focus on fraction as number? Students are also presented with 2 different models. 1 is horizontal and 1 is vertical. We are wondering if and how that might throw off students. We are not doing a 3 read protocol today even though it is a practice we strongly support as a support. We are doing this in order to see how students understand and represent the fractions. If students are not able to see the visual model and the relationship to the number line, than they are not understanding fractions the way we hope they do. In 2nd grade, part of the standard is… different shapes…

- What is the **new student learning** in this lesson?
- Students have already been introduced to 10ths, but it is the 1st time they are seeing them in a real word context. It is also the first time they are comparing fractions with different denominators and numerators.
-
- How will you **assess student learning**? How will you **assess progress** toward your **theory of action**?
- Do students realize that this is a fraction scenario? Turn and talks: are students able to articulate the problem and express their thinking verbally and in their notebooks? Are students able to manipulate 10ths? Number of students that use number line v visual model? --- Will this lesson help them understand decimals in 4th grade? Do students come to a consensus about the fractions being equivalent?

Analysis:

Questions/Wonderings about the *Research Question* and *Theory of Action:*

Questions/Wonderings about the lesson design:

Q: In the lesson prior to this one, students labeled 10cm… Did the use the word 10ths?

A: Students will have fraction bars to create 10ths. That is how they will make their number lines

Q: How much experience have they had using the number line to solve problems with whole numbers?

A Students are very familiar with them. They have been using them for a number of years to solve problems?

Q: Have they had any experience counting unit fractions with the number line?

A: Yes, the four lessons we mentioned earlier.

Q: You said there will be a vertical and a horizontal representation in the problem today..

A: (lost this one)

Phil- I want to compliment you on your approach to standards.

Audience:

Q: How long does this process usually take you?

A: We have been planning this for at least 3-4 months. Once a week for about 2-2.5 hours and then more closer to the lesson for a few months. Some lessons might just take a month or two depending on the goals and the amount of learning needed. Our 2nd grade lesson took a lot of time, but we did a lot of learning beforehand.

Q: Ho any of the lessons are taken from your curriculum and then adjusted and how many do you start from scratch?

A: We use a variety of sources. We use what will support the methodologie and student learning. That is also why the planning time can vary so much.

Q: What do you see in your students learning shifts based on this journey you have been on.

A: Students view themselves as mathematicians. Students are able to explain and defend their thinking. They are willing to make mistakes publicly and learn from each other. They understand that their job is not to be right, but to understand the math and help ensure that all members of the community understand. Students are able to persevere - productive struggle. The other thing is that students are able to make up real life contexts themselves to contextualize problems. Engagement and excitement about math. They love math now. When I have a sub and they have worksheets, they get angry. They love math to the point that they are no longer running out of the classroom - whereas they might be in other contexts. There is a lot of celebration around changing thinking through the process. Students body language demonstrates excitement and engagement with math class.

A few students are newcomers -Karen will translate their responses for the group. These two students will have the lesson question printed for them in Spanish.

Evidence Share-out (data collected during research lesson):

Scripting

Teacher Reflection: This part is the most fun, but maybe not for me. I was pretty impressed with student talk. I was happy to see them change their thinking when discussing misconceptions. The summary at the end that the student came up with was exactly as scripted and I was pretty impressed with that. I noticed that 17/20 used number line to demonstrate their thinking. It seems like they find the number line useful. Only 3 of them used a visual model. I didn't anticipate the impact that students having made their own fraction strips would have on their outcomes. I wonder what impact having store bought fraction strips would have had on the outcome. Not having students exposed to different numerators and denominators in the lesson may have led to confusion in today's lesson and some mislabeling. We didn't go over definitions for numerator and denominator, and I'm wondering what impact that may have had on student understanding and outcome. I'm proud of the work students did and the outcome.

Moises & Mathew: They quickly came to the conclusion that Ms. Cortez ate more. Matthew drew one number line using the same piece. He left out 7/10. When he got to 10/10 the line was not complete. This did not bother him. Students are drawing inferences from the visual model. It works when the visual models are good. When they put their friends' ideas in their books, they did not align the 5/10 and ½ in the number line or bar model, but the bar model was a little closer to alignment.

Erik & Monica: M said we can't know because we need to know the size. M started with a visual model. E started with a number line. M saw that and took out her 10ths. She didn't line up her zero on the grid line. Her ½ didn't line up either, but then she grabbed another half to check placement and noticed that her work was misaligned. E used 10ths from his big to measure out, but they were actually 8ths, so 4 of his 10ths made ½ because they were really 8ths.

Lionel & Carlos: first turn and talk. L ate more because it is a bigger piece. Wait what if Ms.C has a really big sandwich and then the little pieces could be bigger. C used 5ths on number line and wrote the denominator in the numerator spot. L used 10ths. C switched to 10ths and then pulled out the ½. He saw that the two line up and that they were the same. L got confused and did not see that they were eating the same amount. L was stuck on the idea that Silvia stopped at the 5th tick mark of 5/10ths instead of doing all 10 to show that there are 10 equal parts.

Alexander & Arturo: Arturo - they couldnt say because they needed to know how big the sandwiches were. The amount is the same, but one has larger pieces and the other has smaller pieces. On his number line he measured out 10ths but labeled it using 5ths. Alexander measured out and labeled 10ths. When he copied his friends idea, he copied the models side by side instead of one on top of the other.

Daniela & Aldo: Both of them started quickly and were engaged in the conversation. D confused the situation around which sandwich belonged to whom. Both drew a number line with a fraction bar. Both D and A got out the 10ths and the ½ pieces. D rreferred to earlier work with 5ths. SHe started working and marking 5ths. A was working with 10ths. D switched to 10ths. D said one ate more than the other even though her work showed that they had eaten the same. A did not come up with an answer. When the two came back to do their friends idea… Ds summary and As summary...

Silvia & Ramon: S started working right away with a number line. Labeled end points 0 and 1. Marked ½ point. Used 10ths to mark lines. Labeled 9/10 as 1/10. Clarified her marks and realized that both ate the same. Ramon sat for 5 mins looking at the board and his work. He eventually drew a fraction bar and number line and then drew out 10ths and ½. ON the rug he disagreed with Monicas and agreed with the other students.

Salem & Samir: Salem - we dont know if the small ones can fit within the big one. Samir - I think they ate the same. They both drew a visual model and wrote that they ate the same amount. They labeled it ½ and ½. After that Salem used the fraction strips to demonstrate the same idea. Salem said I know they are the same because they line up with each other on the number line. Salems reflection matched the summary.

Camila & Byron: Initial turn and talk: C said that Ms Cortez had eaten more b/c she had five pieces. They were not talking in fractions. Once Monica showed her work, then they switched to representing using fractions. C's model showed equivalent fractions but she said that Leila ate more. Both students built their model up to ½ and 5/10 and did not check to make sure that the parts would create the whole. … On the carpet, C was confident that the two ate the same b/c the sandwich size was the same.

Kelvin & Isabella: Both had a misconception at the beginning. They were confused about who had 5/10 and who had ½ . They both agreed that 5/10 was more. They took a long pause before making a number line. K pulled out 5ths and labeled ⅕ ⅖ ⅗… Isabella used

10ths. The two did not notice that they weren't using the same tools. Isabella didn't line up her pieces well, so 5/10 and ½ did not line up. Both used their notebooks as reference throughout the lesson.

Justin & Genesis: Both got out their whole and drew a number line. G split her line into 5ths. Like her work from yesterday. Both used fraction bars to make their drawings. J used 10ths…. WHen Karen came over G erased all her work and then pulled out the 10ths. SHe spent a long time trying to make all the 10ths fit, but was unable to do so. During the WG time, she was paying ...

Summary of evidence collected:

Post-Lesson Discussion/Debrief Notes:

Scripting

I. **Student Learning & Thinking:** What have we learned about our students, the content, and instruction? Were the lesson goals and learning targets met? How did the lesson design enable student learning and illuminate student thinking? Was new learning embraced and did students engage in productive struggle? Were students engaged and hooked? What might we have done differently?
II. **Research Question & Theory of Action:** What have we learned about our research question and our theory of action? What implications does this have on our next steps? Do we need to revise our theory of action?
III. **Next Steps:** What are our strategic next steps as a team?

Q: Kit. Is the ruler the same size as the whole? Some students used their ruler as the whole.
A: No. It is a tool for organizing their notebooks.

C:I expected students to take out 5 1/10 in order to build their model instead of using the same piece over and over again. I wonder if some students would have been more successful if they had done that.
R: Even if they didn't line up their whole, in earlier lessons they were comparing like, so it didn't have as big of an impact on the end result. I think they used the same piece over and over again because that is how they know to prove, but it only works if the pieces made well.

C: Some students, when they did their own thinking, they did not line up with the grid, but when they wrote down their friend's work, they did.

C: Students are used to the amount as something that you count. Amount here is a lot more complicated in this problem. The amount of sandwich is the question posed. How are students seeing amount in the diagrams that they are using. You want them to get to the point that they understand that amount is length. Lining up is important because we are measuring lengths. The other complexity is that we are counting 1/2s and 1/10s so we are counting different things. So when we compare we have to do something in order to compare. I don't think they are at the point yet where they understand the summary, but they are moving towards it.

C:Compare numerator, compare denominator, same and different, find equivalencies. Where would students have seen equivalencies where bigger is smaller or the same?

C: Usually we just do 2/4, ½, 3/6 for equivalencies, but we really wanted to see what they would do with 10ths because of the lead up to decimals, and because students have done a lot of work with 5 as half of 10 in earlier grades. We did not give them the terminology of equivalent fractions yet b/c

C: at first we thought about using 2/4 and 5/10 but we thought our problem as it was today was more accessible.

Q: Was 10ths limiting? Was it the right choice?
A: I don't think it was limiting. I do think we could have chosen, for the first time being exposed to equivalent fractions, a different number, but I don't think the 10ths were limiting.
A: I think the hand made tools added challenge.
A: I wonder if students prior experience with 10s actually supported students understanding in this lesson. Students may have quickly gone to 5 is half of 10.
A: 2/4 may not have been any more accessible.
A: What numbers get them to think about the thing we want them to think about? We need to see how it plays out in the next few lessons before we really know. - Even numbers often make it easier to partition well because students can easily fold to break down.

Q: Why do you think students didn't just look at the ½ piece and their 1/10 pieces to do their work?
A: It may be because students know that the work needs to be represented and communicated, so they may have gone straight to ways that can be communicated on paper.

Q:?
A: In previous years we may have used the grid lines to focus on, but it is limiting in what fractions students can readily used, and we think it may limit students understanding. The fraction bars and number lines lend themselves to more flexible and deeper understanding.

C: All students that made a visual model drew the line horizontally to split in half.

C: Students were very focused and got to work. I'm wondering if students know they have a set time amount of time and will need to defend their work that means that they are merging exploration and defense work. Is it limiting their exploration time? What is the right balance?
A: The various parts of the lesson might allow for more exploration and that students will be in different places in their work during various parts of the lesson.
A: We have moved away from exploration for its own sake or just using manipulatives…
A: Students did create their own strips and explored the strips and their relationships.

C: Something that we have worked on a lot is the use of manipulatives. Don't over scaffold. Use tools if needed and not if not.

Analysis:
Questions or themes that guided the conversation:

Questions or themes for future discussions:

Expert Commentary Notes:

Scripting (of expert commentary and of discussion afterwards):
Tad Watanabe:

When you have a carefully thought out lesson, you can learn a lot of things.
We often ask questions as teachers that we hope at some point students will ask themselves. The same with tools. We hope that we can take them away at some point and they will still be successful. We learn those kinds of things when we have a carefully planned lesson.

When teachers observe lessons, teachers learn things. In the LS context, with research question and theory of action, we learn something specific. Learning becomes more purposeful.

Students did learn in a previous lesson about equivalencies. 2/2=1, 4/4=1. When you have fractions you can express the same amount in different ways.

The students don't necessarily have difficulty dealing with 10ths. They can also have productive struggle working with fractions with different numerators and denominators.

Challenges looking ahead:

1. The idea of number line: Are students really using number line? It looks more like students used line segment. It seems like when you use number line, you may want to have students put the 2 fractions on the same line as opposed to making 2 lines.
2. Some students said the quantities were the same because it looked like it. Others were going in a different direction, but were struggling to explain. That is our job - to help students do that. What are some of the reasonings we want students to use to explain ½ is the same as 5/10?
3. When you compare fractions with like denominators you basically just count it. With like numerator, you just look at the size of the piece. When they are both different - The purpose of doing this kind of work early in the learning is for students to understand what each does and their relationship to each other - that you have to look at both. Initially I was a little concerned that they have not yet compared like numerator in the unit< but I'm not a big fan of that because students tend to think that they just have to look at the numerator and don't need the denominator to compare. I think in this lesson we really need to emphasize that we have different size units. I think the students are seeing it, but we need to make it explicit.

In the next lesson, using the models of the 2 sandwiches, we might want to ask students to demonstrate a ½ in 5/10 and how do you see 5/10 in ½? Have students use the same number line in the future. Maybe they could use a different color for each sandwich to keep track.

Phil: Notice that Tad's recommendation for the next lesson isn't really a problem. It is an invitation to go deeper into the math and ways of representing.

Q: When you are doing these kinds of set-ups, how do you balance not having the same 3-4 kids always come up and do the work and having issues of status.

A: students are used to the method and we as teachers arethiking about balancing who comes up. We can also be strategic - the lesson might be about many things and we can match who we pick to a variety of strengths. Sometimes there are multiple right ways and was can invite many students into that space. Some teachers share a few reflections from the previous day at the top of the lesson, and this is a space where students can also have a strong presence in the room. I think that the hierarchy becomes a problem with teacher praising the right answer instead of the problem solving. The shift in culture can create a space where students view themselves differently.

Summary of key points:

XII. SUMMARY OF LEARNING

(To be completed AFTER the research lesson)

Summary of learning from the Post-Lesson Discussion & Expert Commentary
We recommend having each member of the lesson study team complete this reflection individually (see template below), before meeting as a team to discuss collectively. Summarize your learnings below after discussing:

What we learned about the mathematical topic / "Core Math" (including the CCSS-M standards and mathematical practices). What we learned about how to teach this content, how our students learn this content, and use of the curriculum to support the content. Include what you would change in the lesson if you were to re-teach whole-class or to small-groups. ***What will we change in our teaching practice as a result?***

Students are capable of manipulating tenths at third grade. This cycle reinforced that TTP methodology aligns nicely with the math practices 1-8. Students can compare fractions with unlike numerators and unlike denominators. They can use tools to represent their thinking and prove their answer. We learned that students are able to recognize a fraction situation when it wasn't obvious. Students can move from concrete to abstract representations on their own.

We would use precise fraction bars as there were many errors when using bars to create number lines. We will make sure they are representing the number line accurately (lining up the number lines). Work on naming the fractions (ex. many said "one and a half" instead of "one half") and which number goes where --> numerators and denominators (they confused 5/10s with ⅕).

What we learned about our research question and theory of action. What we learned about the instructional practices and productive habits intended to support our research question and theory of action, such as whole-class discussion, mathematics journals, etc. ***What will we change in our teaching practice as a result?***

If their tools were precise than tenths wouldn't have been as limiting. Fraction bars not made correctly were the limitation in this problem. We won't know until 4th grade if our theory of action

Implications for our next lesson study cycle (for example: topics to investigate, modifications of our theory of action, modifications of the lesson study process, how we worked together and communicated as a team, vulnerability and courage to teach publicly, etc.) ***What will we change within our team's lesson study cycle as a result?***

We should do another launch to decimals in 4th grade since we want to learn more about theory of action. When we are teaching 3rd grade equivalent fractions we need to make sure the tools are accurate.

Implications for our school (for example: what we want to share with colleagues, implications for school professional development, implications for how math is structured and taught at this school, recommendations for our administration or ILT, etc.) ***What will we bring to our school as a result?***

We should continue have mixed grade levels for the 2nd cycle/semester of lesson study. It's okay to ask questions about the standards and if they are preparing students for math in future years. We need more of Tad's unit progressions to ground our work!

We should think about the number of people at a public lesson and if we can have an in-house our expert commentary. This would provide more scaffolds to new teachers or teachers new to lesson study.

Implications for our district (for example: recommendations to other schools, changes to the SFUSD curriculum ("Core Math"), type or frequency of support needed from central, recommendations to math department re: math content/practice, etc.) ***What will we bring to our district as a result?***

TTP methodology is working really well for our students and should be adopted more widely by the district. Cross-site public lessons are very empowering and creates more buy-in and investment.

Next Steps. What do we plan to do differently? What will we do *short-term (immediate actions)* and *long-term (ongoing or future actions)* as a result of this lesson study cycle? Be specific.

Short term	Long Term

End-of-Cycle Individual Reflection

Individual ID#________________

Please post or distribute this page so that each team member has a copy and can write an individual response, and please include the individual responses with your video and artifacts from your lesson study cycle.

What we learned about the mathematical topic / "Core Math" (including the CCSS-M standards and mathematical practices). What we learned about how to teach this content, how our students learn this content, and use of the curriculum to support the content. Include what you would change in the lesson if you were to re-teach whole-class or to small-groups. ***What will we change in our teaching practice as a result?***

What we learned about our research question and theory of action. What we learned about the instructional practices and productive habits intended to support our research question and theory of action, such as whole-class discussion, mathematics journals, etc. ***What will we change in our teaching practice as a result?***

Implications for our next lesson study cycle (for example: topics to investigate, modifications of our theory of action, modifications of the lesson study process, how we worked together and communicated as a team, vulnerability and courage to teach publicly, etc.) ***What will we change within our team's lesson study cycle as a result?***

Implications for our school (for example: what we want to share with colleagues, implications for school professional development, implications for how math is structured and taught at this school, recommendations for our administration or ILT, etc.) ***What will we bring to our school as a result?***

Implications for our district (for example: recommendations to other schools, changes to the SFUSD curriculum ("Core Math"), type or frequency of support needed from central, recommendations to math department re: math content/practice, etc.) ***What will we bring to our district as a result?***

Next Steps. What do we plan to do differently? What will we do *short-term (immediate actions)* and *long-term (ongoing or future actions)* as a result of this lesson study cycle? Be specific.

Short-term	Long-term