Lesson Research Proposal for 5th Grade Multiplication of a Decimal Number by a Whole Number

1. Title of the Lesson: Multiplication of a Decimal Number by a Whole Number

2. Brief description of the lesson

Students will use previous understandings of multiplying whole digit numbers, estimation and place value to calculate 0.8 x 6.

3. Research Theme

Teach scholars to construct viable arguments and critique the reasoning of others *through note-taking, board work, and student discourse.*

Teach scholars to make sense of problems and persevere in solving them by *teaching mathematics through problem solving.*

4. Unit Goals

Students will understand the meaning of multiplying and dividing a decimal by a whole number. They will understand these calculations by relating them to what they already know about multiplying and dividing whole numbers and their understanding of place value. Specifically, that the value of digits increase by powers of ten when moving to the left, and decrease by powers of ten when moving to the right.

Students will understand the meanings of the calculations of decimal numbers x whole numbers, decimal numbers ÷ whole numbers, and whole numbers ÷ whole numbers when the quotient is a decimal number. Additionally, students will understand how to carry out such calculations, and they will further their understanding of the place value structure of decimal numbers.

Students will be able to use concrete objects, diagrams, and math sentences to find solutions, communicate their ideas to others, and summarize how to do the calculations.

Students will be able to do calculations of decimal numbers x whole numbers, decimal numbers ÷ whole numbers, and whole numbers ÷ whole numbers when the quotient is a decimal number.

5. Lesson Goals

Students will understand the meaning of multiplication of a decimal by a whole number by relating them to what they have already learned about whole number multiplication and about place value, specifically their previous experience multiplying whole numbers and decimals by ten and recognizing that the digits do not change when multiplied by ten, the digit shifts one place to left, increasing it's value ten times.

6. Relationship of the Unit to the Standards

Previous Grade	Current Grade	Next Grade
4.NBT.1 – Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* 4.NBT.3 – Use place value understanding to round multi-digit whole numbers to any place.	5.NBT.A.1 - Recognize that in a multi-digit number, a digit in one place represents10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. 5.NBT.B.7 - Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	6.NS.2 – Fluently divide multi-digit numbers using the standard algorithm. 6.NS.3 – Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

7. Background and Rationale

In previous years, multiplying decimals has been introduced after students have demonstrated mastery of whole digit multiplication. Additionally, students have been most successful when they have practice with estimating reasonable solutions and focusing on place value during operations. This current class has done exceptionally well with the preceding topics: number theory, whole number multiplication, estimation and place value.

It is the team's opinion based on previous experience that students can be left with only a procedural understanding of decimal multiplication if certain aspects are not emphasized during lessons. This tends to happen when both teacher and students focus on the steps or the rule. During the unit the team will regularly stress estimation and place value before and during all operations. The goal is to understand the mechanics of the decimal multiplication algorithm and have a conceptual understanding for it.

The conceptual understanding for the decimal multiplication algorithm will be grounded in the use of estimation and visualisation with a dual number line. The dual number line serves as a tool that lays the foundation for understanding the multiplicative relationship of proportions. Both strategies were used during whole digit multiplication. The dual number line and estimation will continue to be used as students study fraction multiplication and division of decimals and fractions later in the year.

8. Research and *Kyozaikenkyu*

The Common Core State Standards for Mathematics emphasizes that in fifth grade instructional time should focus on three critical areas including "extending division to 2-digit divisors, integrating decimal fractions into the place value system, developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations." The CCSS-M goes on to specify that students in fifth grade should "use the relationship between finite decimals

and whole numbers (i.e., a finite decimal multiplied by an appropriate power of ten is a whole number), to understand and explain why the procedures for multiplying and dividing fractions makes sense" According to the Progressions for the Common Core State Standards in Mathematics written by the Common Core Standards Writing Team, because of the uniformity of the structure of the base-ten system, students use the same place value understanding for multiplying and dividing decimals that they use for multiplying and dividing and whole numbers. The Progression document suggests that before students consider decimal multiplication more generally, they study the "effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand." A challenge for the team is to design a lesson that not only helps students to develop this deep understanding of multiplying with decimals, but also addresses and undeveloped understanding of place value that may exist for some students.

In researching this topic, the team looked at Van de Walle's text, *Elementary and Middle School Mathematics: Teaching Developmentally*. Our study of Van de Walle's text on developmentally appropriate methods for teaching mathematics echoed the Common Core Writing Team. Van de Walle stresses the importance of students having a clear understanding of the underlying foundation of the base ten system. That there exists a "10 - to - 1 relationship between the value of any two adjacent positions". In regards specifically to multiplying decimals, Van de Walle argues that specific rules for decimal computation are not really necessary if computation is built on a firm understanding of place value. Van de Walle warns that by focusing on rote application of rules students lose out on approaches that emphasize opportunities to understand the meaning and effects of operations and are more prone to misapply procedures. Van de Walle also argues that students should become adept at estimating decimal computation before they are able to master paper and pencil computation. He suggests that in a problem such as 3.7 times 4 that students use estimation strategies to discuss the range of reasonable answers. Could the answer be less than 12? Could it be more that 16? This allows students to then solve the answer and judge its reasonableness based on the estimation discussed previously.

After looking at the CCSS, the CCSS Progression Document, and Van de Walle's book on teaching mathematics developmentally, the team next looked at Tokyo Shoseki's Mathematics International (MI) to look at how multiplication was taught and how place value was taught to develop lessons that would lead up to the launch of this unit on multiplying decimals. One important thing we noticed that differs from Common Core is that MI teaches multiplication as multiplicand x multiplier = product. For instance, in our research lesson the story context of 6 cartons of juice each containing 0.8L of juice would be written as 0.8 x 6 = 4.8 and interpreted as 0.8L of juice multiplied by 6 cartons equals 4.8 L. The team also looked carefully at how MI treats place value. In the first unit of MI grade 5, place value is discussed in problem solving lessons in which decimal numbers are multiplied by 10 and 100. Students learn that when a decimal number is multiplied by a power of ten the digits in the product stay the same as the digits in the multiplicand, they simply move to the left how ever many powers of ten it was multiplied by. If multiplied by ten, the digits move to the next larger place value. If multiplied by 100 the digits move two places to the left because 100 is 10x10. By focusing on both whole numbers and decimals, students learn that the underlying structure of the base ten place value system, that each adjacent place value position has a 10 to 1 relationship, applies to decimals as well as whole numbers. This is intended to help students avoid thinking about operations with decimals as some separate branch of math, and rather a continuation of what they have already learned about whole number operations. These two important topics in MI, multiplication of whole numbers and understanding of place value, were important in designing the lessons that lead up to the research lesson, which is the first lesson in a unit on multiplying decimals, adapted by the team from the 4th grade MI unit, "Let's think about multiplying and dividing decimal numbers"

9. Unit Plan

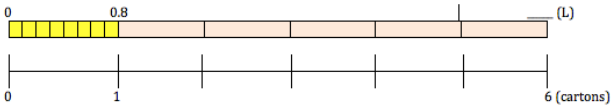
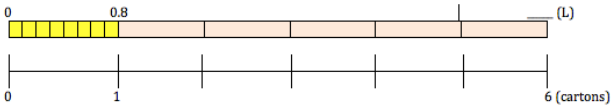
	Let's Think about Multiplying and Dividing Decimal Numbers **Math International** **Grade 4, Chapter 15**	**Assessment**
Lesson 1 **Research Lesson**	**Problem** We bought 6 cartons of juice. Each carton of juice contains 0.8L of juice. How much juice is there altogether? **Anticipated Responses** *Repeated addition *Traditional algorithm with misplaced decimal *Estimation to determine placement of decimal point (.8 is close to 1, and 6 x 1= 6, so the answer is probably 4.8) *Multiply as if they were whole numbers, and then divide the product by 10 **Summary** The product of 0.8 x 6 can be calculated by first making 0.8 10 times as much, then by calculating 8 x 6, and then by dividing the product by 10.	**Exit Slip:** 0.2 x 6 0.3 x 5
Lesson 2	**Problem** There are 7 water jugs each of which can hold 3.6 L of water. If we fill these jugs with water, how much water will there be altogether? **Anticipated Responses** *36 x 7= 252, so 3.6 x 7 = 25.2 **Summary** 3.6 x 7 can be calculated by first making 3.6 10 times as much, then by calculating 36 x 7, and then by dividing the product by 10. **Additional Practice** **IXL** *5.I.3* Multiply a decimal by a one-digit whole number **Multiplication Scoot** Multiply a decimal by a one-digit whole number.	**Exit Slip:** 1.2 x 4 3.7 x 4
Lesson 3	**Practice** Let's solve the same problem from yesterday using the multiplication algorithm: 3.6 x 7	**Exit Slip:** 1.2 x 4 3.7 x 4

	Teacher Guidance 3.6 x 7 1. Write the two numbers by lining up the digits on the right. Disregard the decimal point. 2. Calculate as if you were multiplying two whole numbers. 3. Write the decimal point for the product directly beneath the decimal point in the top number (the multiplicand). **Summary** To multiply decimal numbers, first find an estimate for the solution. Then ignore all decimals and multiply as if they were whole numbers. Reason where to place the decimal based off the estimate. **Additional Practice** **IXL** *5.I.3* Multiply a decimal by a one-digit whole number	
Lesson 4	**Practice** Explain how 0.2 x 4, 0.8 x 5, and 7.5 x 4 can be calculated using the multiplication algorithm. **Teacher Guidance** Let's use estimation to check how reasonable our answers are. **Summary** Today we practiced using estimation to make sure our answer is correct, especially when the product ends in zero.	**Exit Slip:** 0.3 x 2 8.5 x 6
Lesson 5	**Problem** Explain how to calculate 1.8 x 34 using the multiplication algorithm. **Anticipated Responses** *Multiply 18 by 34, then divide the product by 10 *Multiply 1.8 by the ones place first, and then by the tens place **Teacher Guidance** *72 and 54 are the results of what calculations? *How should we decide where to write the decimal point in the product? **Summary**	**Exit Slip:** We are going to give 2.6 m of ribbon to each of 13 people. How many m of ribbon do we need?

	We think about place value to determine where we write the decimal point in the product. **Additional Practice** **IXL** *5.I.4* Multiply a decimal by a multi-digit number	
Lesson 6	**Problem** One meter of an iron bar weighs 1.36kg. How much will 7 m of this iron bar weigh? **Anticipated Responses** *Multiply 136 by 7, then divide the product by 100 *Multiply each place value by 7 **Teacher Guidance** *136 is how many times as much as 1.36? *First make 1.36 100 times as much, then calculate 136 x 7 using the multiplication algorithm. Finally, divide the product by 100. *Explain how 1.36 x 7 is calculated using the multiplication algorithm. **Summary** The product of 1.36 x 7 can be calculated by first making 1.36 100 times as much, then by calculating 136 x 7, and then by dividing the product by 100.	**Exit Slip:** 4.83 x 2 1.25 x 5
Lesson 7	**Practice Problem** Explain how the multiplication algorithm is used in the calculations on the right: 0.25 x 3 = 0.75 2.45 x 4 = 9.80	**Exit Slip:** 0.16 x 4 4.28 x 5

10. Lesson Plan

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Points of Evaluation
1. Posing the Task	Teacher will write question on the board and instruct	What evidence is there that students

We bought 6 cartons of juice. Each carton of juice contains 0.8L of juice. How much juice is there altogether? T: Talk with your table and decide what number sentence we will solve for today's lesson. What are the important numbers? How do we know what operation to do? Expected Student Responses: Our number sentence is 0.8 x 6 Our number sentence is 6 x 0.8 Our number sentence is 0.8 + 0.8 + 0.8 + 0.8 + 0.8 + 0.8 Write on board: *0.8 x 6 = ?* Write on board: *Let's think about how to multiply a decimal by a whole number.* Offer dual number line as a helpful tool in solving the problem. Stress that its use is optional. Post on board: 	students to write the question in their notebooks. Does it matter if we write the problem as 0.8 x 6 versus 6 x 0.8? Which property of mathematics tells us they have the same value (Commutative Property of Multiplication)? What is the relationship between multiplication and addition?	understand the problem? Were they able to highlight key ideas? Do students recognize that 0.8 x 6 represents this situation?
2. Anticipated Student Responses **Response 1:** 0 0.8 1.6 2.4 3.2 4 4.8 L  **Response 2:**		What strategies are students using to solve the problem?

0.8 +0.8 +0.8 +0.8 +0.8 +0.8 _____ 4.8 **Response 3:** 0.8 x 6 _____ 48 My answer needs to be close to 6 x 1, because 0.8 is almost 1. 4.8 is closer to 6 than 48 and 0.48 **Response 4:** 6 x 0.8 = ? (10 times more) ↓ 6 x 8 = 48 (10 times less) ↓ ↓ 6 x 0.8 = 4.8 **Response 5:** 0.8 means 8 of 0.1, so 6 x 0.8 = (6 x 8) of 0.1 = 48 of 0.1 = 4.8 Possible misconceptions: 6 x 0.8 = 48 6 x 0.8 = 0.48	Encourage students who only solve with repeated addition to use dual number line. Ask student to make a ballpark estimate to check the reasonableness of his or her answer. Or ask student to check solution with repeated addition.	
3. Comparing and Discussing **Share 1** Student uses dual number line to determine total liters of fluid for 2, 3, 4, 5, and 6 cartons of juice.	Teacher should refer back to the original equation 0.8 x 6 = ? so students understand that their strategies relate to	When students are explaining their classmates' solutions in their own words, are their responses showing

Share 2 Student uses repeated addition of 0.8 six times to get 4.8 liters of juice. **Share 3** Student multiplies 0.8 x 6 similarly to 8 x 6 = 48. He or she places the decimal in the correct place by reasoning that 0.8 is close to 1. Therefore my solution needs to be close to 1 x 6 or 6. So 4.8 is closer to 6 than 48 and 0.48. **Share 4** Student solves 0.8 x 6 = 4.8 by first making 0.8 ten times larger and calculating 8 x 6 = 48. He or she then makes 48 ten times smaller to get 4.8 liters. **Teacher Guidance** For students who do not explicitly refer to multiplying by 10 and dividing by 10 as a reasoning for why we can use 8 x 6 = 48 to solve 0.8 x 6 = 4.8, ask: (1) What do you multiply 0.8 by to get 8? (2) What do you multiply 48 by to get 4.8?	solving a new type of multiplication problem.	evidence of understanding of the problem and solutions?
4. Summing up the Lesson *To solve 0.8 x 6 = □, it is helpful to know 8 x 6= 48. 8 is 10 times as much as 0.8. `* *Therefore 48 is 10 times as much as□.* **Reflection:** After the summary, students will write a reflection in their notebook. The teacher prompt for this reflection will be "Use what you learned today to solve the problem 0.9 x 5 = ? Explain how you got your answer"		Are students able to correctly solve 0.9 x 5 in their reflection? Which strategies do they use to solve it?

11. Evaluation

Are students able to multiply a decimal by a whole number? How did students use what they know about place value and whole number multiplication to make sense of this new kind of problem?

What evidence was there that the student note taking and teacher board work helped make ideas visible to others, and helped students to communicate their mathematical thinking to others?

12. Board Plan