Grades 3/4 Mathematics Lesson Plan

1. Name of the Unit: Ways of counting and mathematical expressions

2. Goals of the Unit

- Students will be able to utilize mathematical expressions by representing various ways of counting in mathematical expressions or inferring ways of counting from given mathematical expressions. (Calculations)
- Students will be able to utilize a mathematical expression by understanding what each number in the mathematical expression is representing. (Quasi variables)

3. About the Unit

(1) Current state of the students

Because students in Grade 3 learn the standard algorithms for addition and subtraction, they have already studied all the necessary ways of thinking to carry out addition and subtraction calculations with any natural numbers. However, although they have studied the basic single-digit multiplication and division where both the divisor and the quotient are single-digit numbers, they have not had much experience in applying multiplication and division.

In contrast, students in Grade 4 already learned the standard algorithms for multiplication and division. In addition, because they studied "broken line graphs" near the end of the first term, I anticipate that they remember exploring how numbers and quantities change over time.

(2) Mathematical focus of the Unit

As students study the materials in this unit, it is hoped that they will experience the usefulness of mathematical expressions to communicate own reasoning to other people by expressing their own ways of counting in mathematical expressions and inferring other people's ways of thinking through their mathematical expressions. I would also like students to experience the usefulness of mathematical expressions to determine the amount of objects that may be too difficult to actually count by recognizing the generalizability of mathematical expressions based on the ways of counting they express. I want students to learn that mathematical expressions are useful in these ways.

In the first lesson, we will explore a task in which students are anticipated to use a variety of ways of counting. Specifically, we will discuss how to count the number of ○ in Figure 1. In this task, we will not discuss the generalizability of mathematical

Figure 1

expressions. Rather, I want to organize the lesson focused around the idea of expressing own ways of counting in mathematical expressions and interpreting other students' mathematical expressions. In order for students to appreciate how mathematical expressions reflect our ways of counting by examining the diagram, I will have students think about other students' ways of counting from their mathematical expressions or expressing other students' ways of counting as mathematical expressions.

In the second lesson, we will explore a task in which students can think about generalizability of mathematical expressions. However, instead of using very large numbers that will make it almost impossible to actually count, we will use numbers that are small enough so that students can easily verify the results obtained from mathematical expressions by actually counting the objects. By doing this, I intend for children to recognize on their own that the numbers in their mathematical expressions may be generalized. Specifically, we will start with counting the number of ○ in Figure 2. As in Lesson 1, I want students to appreciate the correspondence between mathematical expressions and their ways of counting, and, then have them use their ways of counting and mathematical expressions when the numbers are changed.

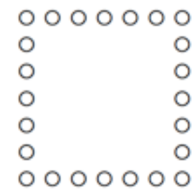


Figure 2

In the third lesson, we will explore a task in which the actual counting is not possible so that students can utilize their realization of generalizability of mathematical expressions experienced in Lesson 2. Specifically, we will use the arrangement shown in Figure 3. However, the discussion of the situation with large numbers will be limited to the second half of the lesson. In the first half of the lesson, just as it was the case in Lessons 1 and 2, I want students to appreciate that their ways of counting are reflected in their mathematical expressions. The reason for organizing this lesson in this manner is because the situation presented in this lesson is not an extension of the situations in the first two lessons. As an extension of the arrangement in Figure 3, we will be exploring the situations in which it is impossible to actually count objects. Thus, I want students to enjoy the situation in Figure 3 first as well. I want students to realize that different situations have their own unique features that they can enjoy.

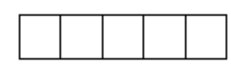


Figure 3

In the fourth lesson, we will explore a task in which students will have to determine the number of ○ in the 10th arrangement by figuring out patterns in the arrangements after they are shown the 1st, 2nd, ... arrangements. The arrangements shown at the

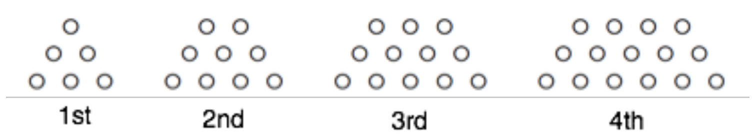


Figure 4

beginning of the lesson are shown in Figure 4. The difference from the first 3 lessons is that students will have to think about generalizability of mathematical expressions from the beginning of the task. Ways of counting are important bases that support the process of generalization.

Another feature of the materials discussed in this unit is that they offer many opportunities to discuss rules of arithmetic calculations (such as the use of ( ) and the order of operations). When we are writing mathematical expressions or making calculations based on mathematical expressions, I want to engage students in active discussions on these ideas.

4. Unit Plan (Tasks & Objectives)

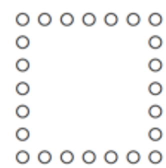
Lesson 1

"Let's think about ways to count the number of ○ in the picture shown on the right." (1 lesson)

- Students will try to express their ways of counting ○ in mathematical expressions and also infer other students' ways of counting from their mathematical expressions.

Lesson 2 [Previous lesson]

"How many ○ are there (arranged in a square pattern)?" (1 lesson)



- Students will express their ways of counting ○ in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will use the mathematical expressions they developed while counting ○ when there are 7 ○ on each side of the square in order to determine the number of ○ when the number of ○ on each side is changed.

Lesson 3 [Today's lesson]

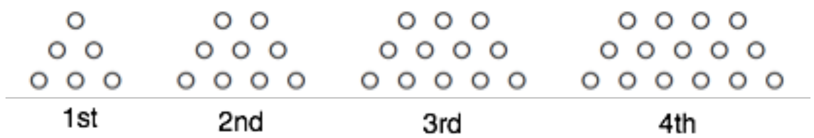
"How many sticks are there all together (when a train of squares are constructed using 1 stick as a side of a square)?"

- Students will express their ways of counting sticks in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will determine the number of sticks when the number of squares is increased using the mathematical expressions they developed while counting the number of sticks when there are 5 squares.

- Students will confirm that they can use their mathematical expressions to determine the number of sticks even when they cannot actually count the number of sticks because the expressions are based on their ways of counting.

Lesson 4

"Here are the 1st, 2nd, 3rd, and 4th pictures (below). How many ○ will there be in the 10th picture?"



- Identify patterns as the arrangements are changed and determine how many ○ there will be in the 10th picture.

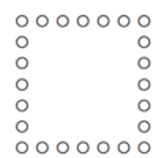
6. Plan for the previous lesson

(1) Objectives

- Students will express their ways of counting ○ in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will use the mathematical expressions they developed while counting ○ when there are 7 ○ on each side of the square in order to determine the number of ○ when the number of ○ on each side is changed.

(2) Flow of the lesson

<table>
<tr><th>● Main student activity
・Anticipated response</th><th>◇ Instructional strategy
◆ Assessment</th></tr>
<tr><td colspan="2">● Own the problem

I made a square by lining up ○ as shown on the right.
How many ○ are there? Let's think about ways to count.
</td></tr>
<tr><td>• 28, thinking the arrangement shows 7 × 4.
• Thinks it is odd that the number is not 28 when counted one by one.
• Knows that there are 24 ○ by counting one by one.

● Think about ways to count ○.

• Wonders why it is not 7 × 4, and re-count the number of ○ on each side.
① To show 24 using multiplication, try to group ○ in 2, 3, or 4.
② Use 7 × 2 for the top and the bottom sides, and 5 × 2 for the left and the right sides.
• 7 × 2 + 5 × 2 = 24 • 7 × 2 + (7 − 2) × 2</td><td>◇ Confirm that there are 24 ○ by counting one by one.

◆ Are students trying to figure out the total number of ○ by relating it to the number of ○ on each side?

◇ For students who cannot represent their ways of counting using mathematical expressions or diagrams, suggest showing how they counted on the diagram first.</td></tr>
</table>

③ Since there are 7 ○ on each side, calculate 7 × 4, then subtract 4 ○ at the vertices because they are double-counted. • 7 × 4 − 4 = 24 ④ Starting from a vertex, group 6 ○ while going around the square clockwise (or counterclockwise) . • 6 × 4 = 24 • (7 − 1) × 4 = 24	
● Think about ways to count ○ while listening to other students' ideas and reporting own ideas. • Tries to figure out ways of counting represented by mathematical expressions that are different from their own. • Does not understand the way of counting represented by a mathematical expression. Tries various ways of counting to see if they match the mathematical expression. • Understands the way of counting by listening to other students' explanations. • Thinks about which way of counting is better.	◇ Encourage students to explain their ideas while going back and forth between their mathematical expressions and the diagram. ◇ Ask students to think about which is the best way to count/mathematical expression. ◆ Are students thinking about merits of mathematical expressions from the perspective that they can be used even when the number of ○ on a side is changed? ◇ If a student points out a mathematical expression or a way of counting can be adapted when the number of ○ on a side is changed, ask "Let's see if we can find the total number of ○ when there are four ○ on each side," to re-examine the mathematical expression from that perspective.
● Using a mathematical expression developed when there were 7 ○ on a side, determine the total numbers of ○ when there are 4 and 10 ○ on a side. • Tries to use the same way of counting instead of using a mathematical	◇ If no students points this out, ask "Now you have picked a mathematical expression you think is the best. Let's use that mathematical expression to figure out the number of ○ when there are 10 ○ on a side."

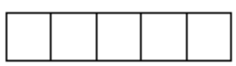
expression. • Tries to use a mathematical expression, but cannot figure out how. Re-derive a mathematical expression based on the way of counting. • Think about which number in the mathematical expression (obtained when there were 7 ○ on a side) should be changed. • Supposes that a mathematical expression that uses "7" is easier to adapt, and replaces "7" in that mathematical expression with different numbers. ● Have students write a journal entry. • Even when the number of ○ on a side is changed, I could use the same way of counting. • Even when the number of ○ on a side is changed, I could figure out the total number by replacing the "7" in the mathematical expression with a new number.	◆ Are students choosing mathematical expressions with "7" as an expression that can be adapted when the number of ○ on a side is changed? ◇ By asking the reason for selecting a mathematical expression, help students share the understanding of the merit of mathematical expressions that uses the number of ○ on a side.

7. Plan for today's lesson

(1) Objectives

- Students will express their ways of counting sticks in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will determine the number of sticks when the number of squares is increased using the mathematical expressions they developed while counting the number of sticks when there are 5 squares.
- Students will confirm that they can use their mathematical expressions to determine the number of sticks even when they cannot actually count the number of sticks because the expressions are based on their ways of counting.

(2) Flow of the lesson

● Main student activity ・Anticipated response	◇ Instructional strategy ◆ Assessment
● Own the problem	
I made a train of 5 squares using sticks of the same length.  How many sticks did I use? Let's think about ways of counting the sticks.	
• Verifies that there are 5 squares connected together. • Knows that he/she must find the total number of sticks. • Knows that there are 16 sticks altogether. • Notice that there is a simpler way to count the ticks instead of one by one. ● Think about ways to count the sticks. ① By using a table or drawing a diagram as the number of squares is increased, explore the number of	◇ Count the sticks one by one to confirm that there are 16 sticks. ◆ Are students counting the sticks by corresponding it to the number of

sticks in relationship to the number of squares. (a) Notices that the number of sticks increases by 3 when 1 square is added and writes a mathematical expression. • $4 + 3 \times 4 = 16$ • $4 + 3 \times (5 - 1) = 16$ (b) Notices that the number of sticks is increasing by 3 but cannot write a mathematical expression. ② By arranging sticks or drawing a diagram, think about ways to count the sticks more easily. (c) Notices that the number of sticks increases by 3 when 1 square is added and writes a mathematical expression. • $4 + 3 \times 4 = 16$ • $4 + 3 \times (5 - 1) = 16$ (d) Each square has 4 sides, so calculate 4×5. Then, since the sides between two adjacent squares are double counted, subtract the number of overlapping sides, 4 (or $5 - 1$), from the product. • $4 \times 5 - 4 = 16$ • $4 \times 5 - (5 - 1) = 16$ (e) Since there are 5 sticks at the top and 5 sticks at the bottom, 5×2. The number of vertical sticks is $5 + 1$. • $5 \times 2 + 6 = 16$ • $5 \times 2 + (5 + 1) = 16$. ● Think about ways to count the sticks while listening to other students' ideas and reporting own ideas.	squares or by determining other units (such as there are two groups of 5 horizontal sticks at the top and the bottom and 6 vertical sticks)? ◆ Are students using the number of squares in their mathematical expressions?

• Tries to figure out ways of counting represented by mathematical expressions that are different from their own. • Does not understand the way of counting represented by a mathematical expression. Tries various ways of counting to see if they match the mathematical expression. • Understands the way of counting by listening to other students' explanations. • Thinks about which way of counting is better. ● Using the strategies discussed when there were 5 squares, students will think about how many sticks will be used when there are 50 squares. • Since 50 is 10 times as much of 5, the number of sticks will also be 10 times as many, or 160. • Based on the ways of counting, explain that, even though 50 is 10 times as much of 5, the number of sticks will not also become 10 times as many. • Tries to use a different strategy from the one he/she used in the previous problem. • Cannot decide which number in mathematical expression should be replaced with "50." • Thinks that mathematical expressions which include "5" can be adapted easily.	ideas while going back and forth between their mathematical expressions and the diagram. ◇ Ask students to think about which is the best way to count/mathematical expression. ◆ Are students thinking about merits of mathematical expressions from the perspective that they can be used even when the number of squares is changed? ◆ Are students verifying that mathematical expressions can be used even when the number of squares is increased based on the ways of counting the expressions represent? ◇ If a student points out a mathematical expression or a way of counting can be adapted when the number of squares is increased, ask "Why can we use mathematical expressions even when the number of squares is increased?" ◇ If no students points out the merit of mathematical expressions that they can be used even when the number of squares is increased, ask "Let's figure out how many sticks will be used if there are 50 squares." ◆ Are students choosing mathematical expressions with "5" as an expression that can be adapted easily? ◇ By asking the reason for selecting a mathematical expression, help students share the understanding of the merit of mathematical expressions that uses the number of squares in them. ◇ If there are students who think the number of sticks will be 10 times as

● Have students write a journal entry. • I learned that even when it is very difficult to count the number of sticks, we can use mathematical expressions to find the total numbers if we use the number of squares in the expressions. • I thought it was a good idea to use the number that can be found first in a mathematical expression.	times as many, have students verify whether or not the correspondence between the number of squares and the number of sticks is indeed such a relationship.