Lesson Research Proposal for Fourth Grade Fractions

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Acorn Woodland Elementary School, Hanna Sufrin's class
Instructor: Hanna Sufrin
Lesson plan developed by: Hanna Sufrin, Julie Kailath, Billy Tang,
Sarah Horwitz, Kari Laux, Jana Morse, Shelley Friedkin

1. Title of the Lesson: Changing 7/3 to a mixed number

2. Brief description of the lesson

Students will explore the relationship between improper fractions and mixed numbers, taking special notice of the whole within improper fractions.

3. Research Theme

Research Theme:

Deepening Conceptual Understanding through a problem solving approach with a specific focus on the operations and algebraic thinking.

Our Theory of Action:

If teachers apply a Teaching Through Problem Solving based approach (TTP), then students will deepen their conceptual understanding of mathematics. This will result in students being able to apply their learning in multiple contexts and to justify their thinking by utilizing or making connections between multiple representations. Conceptual understanding will also support procedural fluency, as students make connections and see patterns between content. Students will understand and apply algorithms effectively as a result of this conceptual foundation. In order for this type of learning approach to occur, classrooms must be socially and emotionally safe where mistakes are valued. Students must work collaboratively to develop skills of productive struggle, problem solving, communication and stamina. Students must develop a growth mindset. Student discussion and journals serve as a way to formatively assess students' mathematical understanding.

4. Goals of the Unit

This unit is an exploration of mixed numbers and improper fractions. The goals include student understanding that fractions can express amounts greater than one. They will learn to express fractional amounts greater than one both as mixed numbers and as "improper" fractions. Finally, they will identify the relationship between mixed numbers and improper fractions, by discovering that both express the same number of wholes and fractional parts, but in two different ways. Specifically, as Tad Watanabe points out, "we can tell easily how many unit fractions (are in an improper fraction)...on the other hand a mixed number can tell us more quickly about how big that number is (in relationship to whole numbers, which we are much more familiar with).

5. Goals of the Lesson:

Our goal for the lesson is for students to represent an improper fraction as a mixed number through either a fraction strip tape model, a number line, or decomposition. Students will be able to pull fractional wholes "3/3" out of an improper fraction "7/3" while writing the remaining "⅓"

6. Relationship of the Unit to the Standards

Related prior learning standards	Learning standards for this unit	Related later learning standards
CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/*b* as the	CCSS.MATH.CONTENT.4.NF.A.1 Explain why a fraction *a*/*b* is equivalent to a	CCSS.MATH.CONTENT.5.NF.A.1 Add and subtract fractions with

quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. CCSS.MATH.CONTENT.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. CCSS.MATH.CONTENT.3.NF.A.2.A Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. CCSS.MATH.CONTENT.3.NF.A.2.B Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. CCSS.MATH.CONTENT.3.NF.A.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. CCSS.MATH.CONTENT.3.NF.A.3.A Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.	fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. CCSS.MATH.CONTENT.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. CCSS.MATH.CONTENT.4.NF.B.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. CCSS.MATH.CONTENT.4.NF.B.3.A Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. **CCSS.MATH.CONTENT.4.NF.B.3.B** **Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*.** CCSS.MATH.CONTENT.4.NF.B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. CCSS.MATH.CONTENT.4.NF.B.3.D	unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* CCSS.MATH.CONTENT.5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*. CCSS.MATH.CONTENT.5.NF.B.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* CCSS.MATH.CONTENT.5.NF.B.4

CCSS.MATH.CONTENT.3.NF.A.3.C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*. CCSS.MATH.CONTENT.3.NF.A.3.D Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. CCSS.MATH.CONTENT.4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. CCSS.MATH.CONTENT.4.NF.B.4.A Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*. CCSS.MATH.CONTENT.4.NF.B.4.B Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* CCSS.MATH.CONTENT.4.NF.B.4.C Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* Understand decimal notation for fractions, and compare decimal fractions. CCSS.MATH.CONTENT.4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[2] *For*	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. CCSS.MATH.CONTENT.5.NF.B.4.A Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).* CCSS.MATH.CONTENT.5.NF.B.4.B Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. CCSS.MATH.CONTENT.5.NF.B.5 Interpret multiplication as scaling (resizing), by: CCSS.MATH.CONTENT.5.NF.B.5.A Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the

	example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. CCSS.MATH.CONTENT.4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*. CCSS.MATH.CONTENT.4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	indicated multiplication. CCSS.MATH.CONTENT.5.NF.B.5.B Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. CCSS.MATH.CONTENT.5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. CCSS.MATH.CONTENT.5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1 CCSS.MATH.CONTENT.5.NF.B.7.A Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division*

		to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. CCSS.MATH.CONTENT.5.NF.B.7.B Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4*. CCSS.MATH.CONTENT.5.NF.B.7.C Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

7. **Background and Rationale**

This year, the fourth grade math team has made a significant shift from direct instruction of new content to an inquiry or discovery model of instruction. Last year, the direct model of instruction was guided by the modules of Eureka Math and fraction instruction began by explicitly teaching decomposition of non-unit fractions into number bonds. With the Teaching Through Problem Solving framework, students are given the opportunity to make sense of fractions greater than a whole number before being taught the specific algorithms such as decomposition and converting mixed numbers into improper fractions and vice versa.

As part of Teaching Through Problem Solving, teachers are using both Japan Math curriculum and Eureka to inform the progression of their units. K-2 at our school have fully taken on the Japanese curriculum and are using _____.

Our school is committed to deepening student conceptual understanding in mathematics and have been doing so through this inquiry based model. K-2 have already fully taken on the Common Core aligned version of the Japanese curriculum. While this is not available for 3-5, upper grade teachers are still using a teaching through problem solving lense in their instruction. They consult the standards, Eureka, as well as the Japanese curriculum to inform their teaching. In developing this unit, the planning team did the same. We wanted to stay as true as possible to the progression of the Japanese unit, but realize we need to factor in the mathematical background of our students as well.

Prior to the introduction of Common Core, math instruction with fractions emphasized procedural fluency over conceptual understanding. For example, teachers may have taught students the multiplicative relationship of equivalence without justifying why that relationship exists with an area model or linear model. The goal of choosing an inquiry based approach is for students to discover that multiplicative relationship by creating multiple representations of equivalent fractions through the area model, partitioning halves into fourths or eighths to represent 1/2 , 2/4, and 4/8 in the same fraction strip.

The focus of this research lesson questions the efficacy of teaching a procedural method of converting a mixed number to an improper fraction. Many teachers learned a strategy/procedure by memorizing three steps: 1) multiply the whole number with the denominator, 2) add the product by the numerator, and 3) use the sum (from 2) as the numerator and the denominator of the fraction part of the mixed number as the denominator of the improper fraction. Without understanding that each whole number represents a complete fraction part (eg. 1 = 3/3), students will not develop a conceptual understanding of why this procedure works. Instead of teaching the procedural method, our research lesson aims to have students prove why a mixed number and an improper fraction are equivalent.

The topic we have chosen for our research lesson is the relationship between a mixed number and an improper fraction. The rationale for choosing a lesson with the culmination of different ways to express fractions greater than one is rooted in the belief that students should grapple with mixed numbers and improper fractions interchangeably. This unit introduces fractions greater than one early in order to solidify student's understanding of fractions as numbers. Mixed numbers can be decomposed and represented as fractional units and fractional units can be rearranged as mixed numbers. This conceptual understanding allows students to manipulate fractions at a much deeper level than if they are only exposed to only the procedure of conversion.

Rationale for a "naked" prompt: This lesson falls toward the middle of the unit, after many days of related instruction. We theorize then that students are ready to solve a problem without context and moreover that this decontextualized math problem may support students in thinking beyond one specific model.

8. Research and *Kyozaikenkyu*

General Progression and Understandings

In studying the Common Core Standards and the Mathematics Framework for California Public School (2013), we learned that the basis for fraction concept understanding happens in grade three. It is in third grade that a student first grapples with the concept of a unit fraction. This is essentially asking students to familiarize themselves with a new number system. Though the symbols are similar, up until now, they have only dealt with whole numbers. In third grade, students begin to understand that the denominator signifies the unit or the size of the fractional part, and the numerator signifies how many pieces of the fractional part you have. Key models such as number lines and fractions strips are used to build fraction number sense. These are the basic building blocks of fractions.

Third grade is also where students begin to compare fractions in certain cases. Fraction comparison in third grade only includes fractions with either the same numerator or the same denominator and always within the same size whole. They explore visual models to help them compare.

The topic of equivalence begins in third grade, but is explored much more deeply in fourth. Our research helped to clarify what is so hard about equivalent fractions. While reading Van de Walle, we saw how important understanding the meaning of equivalent fractions is. It is hard for students to wrap their minds around the idea that ½ and 4/8 can represent the same value. Solidifying this understanding before teaching any procedures for finding equivalence is critical.

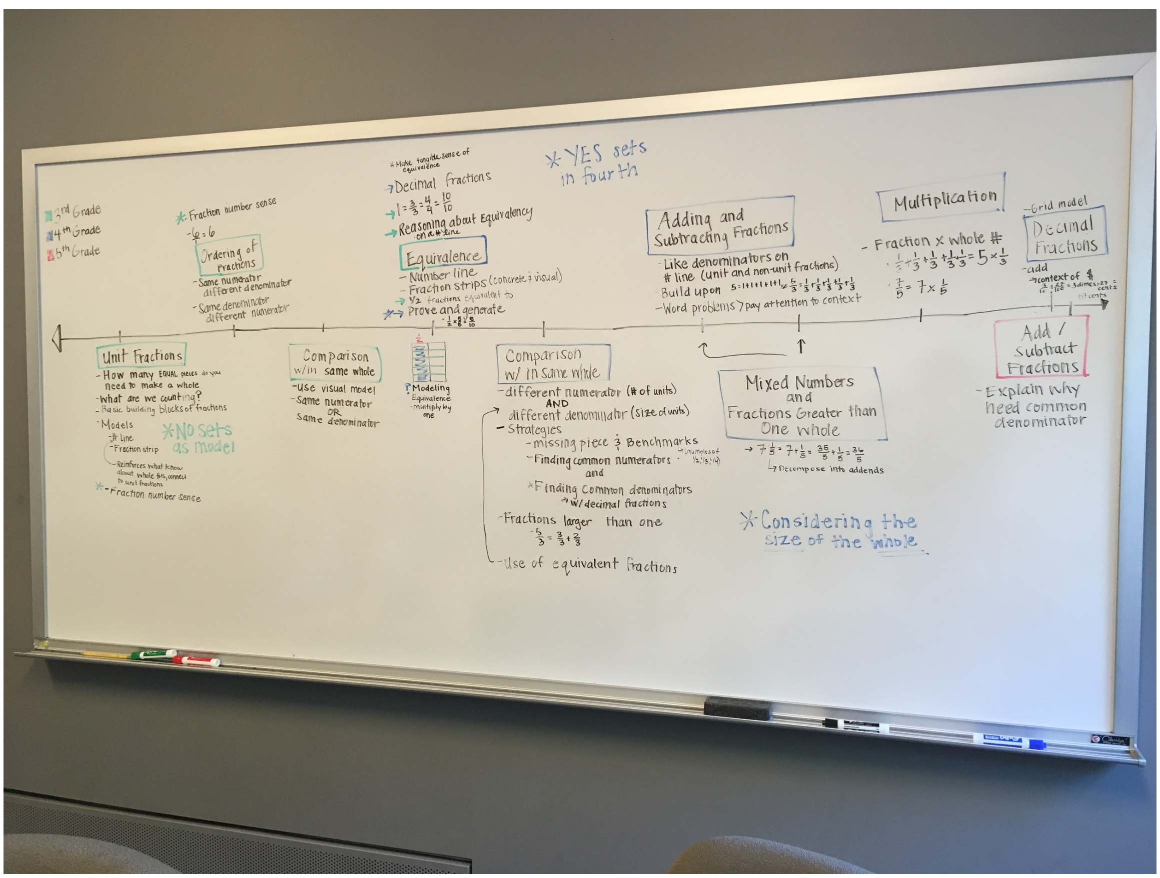
The topic of vocabulary and when to introduce different words in our unit came up in our discussions and also in our Van De Walle reading. The vocabulary we discussed included mixed numbers, proper and improper fractions, equivalence, and compose and decompose when describing strategies. With regard to the term "improper fraction" we discussed Van De Walle's assertion that the term "can be a source of confusion as the word improper implies that this representation is not acceptable, which is not the case at all - in fact, it is often the preferred representation in Algebra." Van De Walle suggests instead to try "not to use this phrase and instead use 'fraction' or 'fraction greater than 1.' But as he acknowledges, the term "improper fraction" may be familiar to students so it's important to share with students "that it is really not improper to write fractions greater than 1 as a single fraction." Another issue we encountered when comparing mixed numbers to "fractions more than one," was the implication that only an improper fraction is a *fraction* more than one. Isn't a mixed number also a fraction? And yet, according to the standards "...leading to answers in the form of fractions or mixed numbers," implies mixed numbers aren't 'fractions."

We also discussed that much like Teaching Through Problem Solving seeks to teach skills in context, so too should we teach math vocabulary in context, especially once a concept is understood. According to a mathematics teacher educator, Tyminski, in the NCTM article, "What's the Big Deal About Vocabulary?", he believes "vocabulary terms should be introduced to students through active engagement in mathematics (TTP) where possible and that students should be allowed and encouraged to find ways to describe the phenomenon they are interacting with in their own words. The teacher's role then is to help students connect the formal mathematical vocabulary with their current understanding of the idea or concept. As a result of our research and subsequent conversations, the decision to introduce relevant vocabulary after a concept has been taught was addressed in our unit planning.

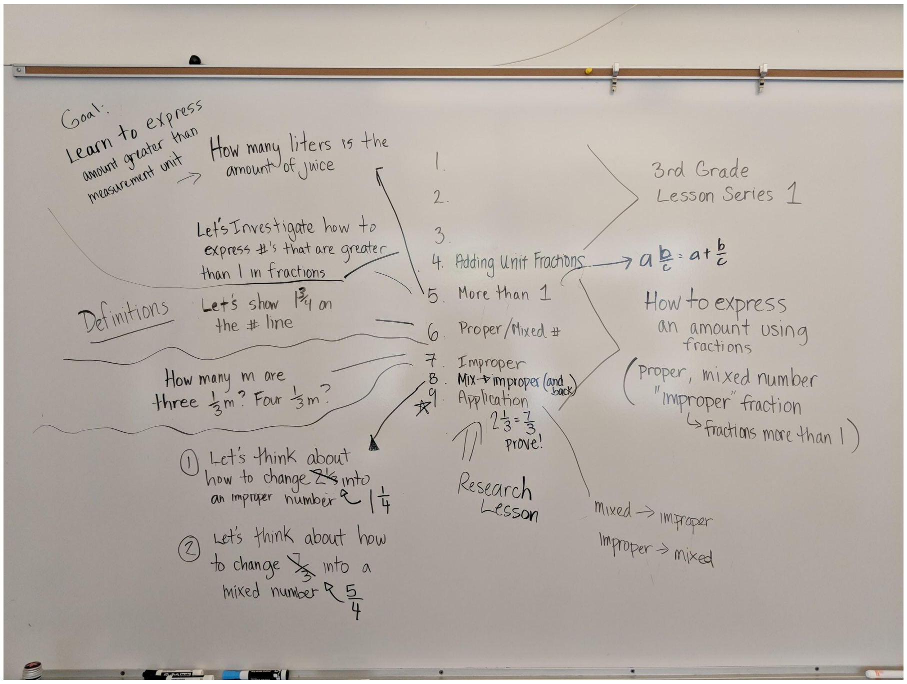
During our research on fractions and concept progression, we discussed when it would be best to introduce improper fractions and mixed numbers. Typically, this concept is taught later in a fraction unit. Most curriculums the team has experienced treat mixed numbers and improper fractions as different from fractions less than one and harp on solidifying the meaning of these (implied strange) numbers. Using a Japanese textbook as one resource for concept progression, we decided it made more sense to address mixed numbers and improper fractions early on because by helping students to see that 1 ¼ is simply 1 + ¼ or 4/4 + ¼ or 5/4 would help students connect to their prior learning of unit fractions. As Tad Watanabe stated, “Helping students to see that 1 ¼ is simply 1 + ¼ or 4/4 + ¼ or 5/4 is simply the extension of the basic meaning of fractions, 5/4 is composed of 5 one fourth units.”

Another important focus in our research and discussions was the importance and challenge of number lines. We came to understand that number lines are not simply an optional strategy to model student thinking, but an essential way to ground fraction concepts within the number system. It is a fundamental mathematical understanding that fractions are numbers which can be used to represent quantities of any size, most importantly those quantities *between* whole numbers.

The data from the pre-test showed that representing fractions on a number line was challenging for the students, a finding supported by team member’s own classroom experience. We decided this might be due to a shallow understanding of what a fraction represents. We also decided that this meant our unit plan should incorporate more frequent use of number lines to help ground fractional amounts and highlight the notion that not only can fractions represent parts of a whole but that too they are numbers that can be represented on a number line.



9. Unit Plan



#	Date	TTP Problem	New Learning	Summary	Standards addressed
1	1/16	Find the length of the mystery strip ⅓ , ½, ⅕	Students will become aware that fractions can be seen in students' everyday lives. Students will understand that fractions are used to express an amount obtained as a result of equal partitioning, and are used to express quantities less than 1 (only unit fractions).	When a whole is made into equal parts, a unit fraction is one of the equal parts.	CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. CCSS.MATH.CONTENT.3.NF.A.2 Understand a fraction as a number on the number line; represent fractions

2	1/17	Find length of the mystery strip ⅖, ⅔	Students will understand that a fraction can be considered as a collection of unit fractions. Students will know fraction notation.	Unit fractions can be combined to make non-unit fractions	on a number line diagram.
3	1/18	How do we write/ show a fraction on a number line? Make ¾ Find the length of other table's mystery strips	Students will become aware that a fraction can be put on a number line.	The number on the top is the numerator. It shows how many unit fractions you have. The number on the bottom is the denominator. It shows how many total segments the whole was broken into.	
4	1/19	Practice Day	Students will add unit fractions with like denominators	When adding fractions, the denominator is the size of the piece, you add the numerator because it tells you how many of that size piece you have.	CCSS.MATH.CONTENT.4.NF.B.3.A Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
5	1/22	Investigate how to express numbers that are greater than 1 in fractions	Students understand and represent fractions more than one using a mixed number ("one and three fourths" 1 ¾, 1 + ¾)	You can use a whole number and fraction number ("mixed number") to represent a fraction more than one.	*
6	1/23	Express 1 ¾ on a number line	Students will understand the concept of "proper fraction" and "mixed number"	A fraction whose numerator is less than the denominator is a proper fraction. A fraction that is	*

				made up of a whole number and a proper fraction is a mixed number.	
7	1/24	Practice Day with Models and Number Line	Students will explore and practice use of linear models and the number line using mixed numbers	We can show whole numbers and fractional parts, together as mixed numbers, on the number line and in our models.	CCSS.MATH.CONTENT.3.NF.A.3.C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers: *Each b fractional units, 1/b is equal to 1. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.
8	1/25	How many meters are three 1/3m? Four 1/3m?	Students will understand the concept of "improper fraction" Students will be reminded that fractions are equivalent to whole numbers *use of the number line / fraction strip model	A fraction whose numerator is the same or greater than its denominator is an improper fraction.	CCSS.MATH.CONTENT.3.NF.A.3.C Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.
9	1/26	RESEARCH LESSON: Can you write 7/3 as a mixed number? How do you know?	Students will change an improper fraction into a mixed number.	To write an improper fraction as a mixed number and find how many wholes are there and how many parts are left.	CCSS.MATH.CONTENT.4.NF.B.3.B Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction

					model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8*.
9	1/29	Investigate a range of fractions (1/2, 2/4, 5/10) that are equivalent on the same number line.	Students recognize proper fractions that are equivalent as showing the exact same amount, despite different fraction sizes.	The same fractional amount may be expressed in different ways depending on the size of the fractional parts.	CCSS.MATH.CONTENT.4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
	1/30	How many ⅛ are the same fractional amount as ¾?	Students generate equivalent fractions using visual models, including the number line, to prove equivalence.	We can find equivalent fractions by making a number line that shows how different numbers of different fractions can actually be the same amount.	CCSS.MATH.CONTENT.4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
10	1/31	Which is bigger, ½ or ¼? Which is bigger, ⅔ or 2/6? Why?	Students understand how to compare fractions with like numerators.	To compare fractions we have to consider the size of the denominator before comparing the numerators. The greater the denominator, the smaller the size of	CCSS.MATH.CONTENT.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.

				the fraction.	Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
	2/1	Daniela and Adrian were eating burritos. Daniela ate ⅓ of her burrito. Adrian ate ⅔ of his burrito. Adrian says Daniela ate more than him. Could this be true?!	Students understand that comparisons are valid only when the two fractions refer to the same whole.	You always have to know the size of the whole that fractions are a part of if you’re comparing them.	CCSS.MATH.CONTENT.4.NF .A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
11	2/2	Lupita used $3/5m^2$ of cardboard and Heidi used $4/5m^2$. How many m^2 of cardboard did they use altogether?	Students will add proper fractions leading to a sum that is improper.	We can add fractions just like we add whole numbers if they have the same denominator.	CCSS.MATH.CONTENT.4.NF .B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
12	2/5	There is 1 ⅖ kg of sugar. If you use ⅘ kg of the	Students will subtract proper fractions from mixed numbers.	When subtracting fractions that have the same	CCSS.MATH.CONTENT.4.NF .B.3.C

		sugar, how many kg will be left?		denominators, just subtract the numerators and leave the denominators as they are. n	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
13	2/6	Show how to calculate 2 ⅗ + 1 ⅕.	Students will add mixed numbers.	We can add mixed numbers by first changing them into improper fractions OR we can add the whole numbers and then the fractions and combine them.	CCSS.MATH.CONTENT.4.NF.B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
14	2/7	Show how to calculate 2 4/5 - 1 ⅗.	Students will subtract mixed numbers from one another.	We can subtract mixed numbers from other mixed numbers by changing them into improper numbers OR we can do whole numbers and the fractional parts separately.	CCSS.MATH.CONTENT.4.NF.B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
15	2/8	Write a multiplication sentence whose product is 5/4.	Students will understand the fraction a/b as a multiple of 1/b.	A fraction can be expressed as a multiple of a unit fraction and a whole number; by multiplying you get that many of the unit fractions.	CCSS.MATH.CONTENT.4.NF.B.4.A Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation*

					5/4 = 5 × (1/4).
16	2/9	Ms. Sufrin ran ⅘ of a mile every day this week. What distance did Ms. Sufrin run this week?	Students will multiply fractions by whole numbers.	We can multiply a fraction by a whole number if the fraction is repeating, just like when we multiply whole numbers that are repeating.	CCSS.MATH.CONTENT.4.NF.B.4.B Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

10. Design of the Unit and Lesson

Students coming into this unit did not have a strong understanding of unit fractions. We administered a pre-assessment and though more than 80% of students were able to represent unit fractions using a linear bar model, only 18% of students were able to place fractions greater than one on a number line. This significant discrepancy indicates that students may not have a strong conceptual understanding of fractions as numbers even though they understand how to represent a fraction as an equal part of a whole. This finding is not surprising given that historically the emphasis in the U.S. math education system has focused on just one construct of fractions - parts of a whole. As Van De Walle states, "Understanding fractions means understanding all of the possible concepts that fractions can represent. One of the commonly used meanings of fraction is part-whole, including examples when a part is shaded. In fact, the part-whole is so ingrained in elementary textbooks as the way to represent fractions, it may be difficult for you to think about what else fractions might represent. Although the part-whole is the most used in textbooks, many who research fraction understanding believe students would understand fractions better with more emphasis across other meanings of fractions." He goes on to define the other meanings of fractions or fraction constructs as Part-Whole, Measurement (unit fraction concept), Division, Operator, and Ratio.

Once we were able to interpret the fraction pre-assessment results using our newfound knowledge gained through research, we made the decision to begin the fraction unit by spending the first three lessons using the Mills College Fraction Tool Kit, lessons grounded in measurement to help students see fractions as a number. We were aware of the fact that the concepts taught in this lesson series addressed third grade standards but were confident this focus would be essential to building the foundation for fraction understanding in 4th, 5th and beyond. The following two reflections reinforced our theory that it was important to start with this lesson series:

Allison: My AHA! moment was when on the strip we could put about 2 of the ⅓ and then it's ⅔ and we add one more of the ⅓ it was 3/3 of a meter.

Adrian: My AHA! moment was that when there's like ⅕ + ⅕ you change the numerator but you don't change the denominator.

11. Research lesson

Anticipated Student Responses:

- Bar Model: Students compose by drawing seven ⅓ unit fraction bars, then group into two groups of 3/3 and ⅓
- Number Line from unit fractions: Students compose by drawing seven intervals of ⅓ on a number line, then group to show 2 wholes and ⅓
- Number Line recognizing equal group-whole number pattern: Students recognize and label the wholes (3/3 =1, 6/3 =2) and the additional fraction (⅓)

Anticipated Misconceptions:

- Bar Model: Students compose 3/3 into one whole and conclude that 1 4/3 is the correct mixed number.
- Students use intervals on number lines and fractional amounts on bar models other than thirds.
- Students create wholes and intervals that are not proportional
- Students flip/ confuse 7/3 with 3/7

LESSON INTRODUCTION:

Quick warm up with students to get math juices flowing.

- Fast Facts at Tables
- Extra time: Fast Fact Songs

Class Chant

Reading of student reflections from the day before.

- Teacher highlights definitions of an improper fraction (a fraction greater than one) as well as a mixed number
- A reflection that helps define these terms will be shared
- Teacher will then ask a students to read from the shared class poster of terms (mixed number and improper fraction

POSING THE TASK:

Can we turn 7/3 into a mixed number? How do you know?

Student Work Time (7 minutes)

- students solve the question in their journals

Teacher is circulating, identifying student methods and ideas, as well as student misconceptions.

Teacher addresses misconceptions 1-1 for any misconceptions demonstrated by 1-3 students.

Students will be given time to turn and share with a neighbor (3 minutes)

Comparing and Discussing (Boardwork)	**Potential teacher questions/prompts:**	Assessment:
(If present in at least 5 student responses): Teacher has student share who can use fraction bars to defend 1 and 4/3 *Other students avidly debate: no, there is another whole in 4/3!*	*Misconceptions:* **if only a few students, teachers will address with students during independent work*	
Teacher has student share who composed using a fraction strip made up of 7 of ⅓ direct her to make this strip on the board. Composition (Manipulative) - student chooses amongst unlabelled fraction pieces and places unit fraction strip pieces of 1/3 reach 2 and ⅓ - they count the thirds to find 7/3 Student explains how the model proves the mixed number of 2 ⅓.	How should we label each piece? What is this fractional piece showing? (pointing to the ⅓) T draws ⅓ above of each piece (T draws unit fraction equation below fraction bars: ⅓ + ⅓)	Are students showing excitement and agreement? Are students able to identify each whole?
Teacher has student share who composed with a number line, asking for student directions for how to use number line as proof. Composition (Number line) - student counts and labels the wholes (3/3 =1, 6/3 =2) and the additional fraction (⅓) - shows that this is 7/3 AND 2 ⅓ Student explains how it shows that 7/3 can be a mixed number.	So you're combining the unit fraction ⅓? How many times(bring in addition equation) ? So you're saying that 3/3 equals 1 whole? So how many thirds is this? (Pointing to 2 wholes/6/3) Hm! Interesting! Is 3/3 one whole? What about 6/3? How did __ know 6/3 is two?	Are students able to identify the second whole? Is the group showing confusion or agreement?
Teacher has student share who used numbers to decompose:	Can this be? Does this make sense?	Are students

Decomposition (Equation) - 7/3 = 3/3 + 3/3 + ⅓= 1+1+⅓= 2⅓ Return to models to see if they show this equation. Class comes to the consensus that 7/3 = 2⅓. Discuss: why are they equal?	Where are the 3/3 in our other models? Where is the 1? 2? Where is the ⅓? Can we find the wholes in each of our models? Can other improper fractions also be written as mixed numbers?	able to connect the different models? Can the group explain why this is true? And apply it to a rule about improper fractions and mixed numbers? (building toward summary)
Summing Up Summary: To write an improper fraction as a mixed number, we find how many wholes are there and how many parts are left over.	What did we prove today about improper fractions and mixed numbers?	Do many students raise hands to offer a contribution to the summary? Do they show support/agreement with idea shared?

12. Evaluation

- How do students respond to the naked prompt? Is it difficult for some to find an entry point without the context?
- Is the linear model useful in this case? Does it deepen student understanding of fractions greater than one?
- Does it seem like students are ready for fractions greater than one? Are they drawing on their knowledge of unit fractions to help them understand?
- Do students see the whole? Better yet, do they see the multiplicative relationship between number of fraction parts and number needed for one whole, two wholes, three wholes etc?

13. Board Plan

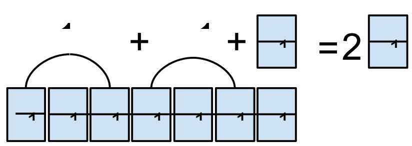
Prompt: Let’s write 7/3 as a mixed number!

Student #1: Composes from unit fraction strips:

What equation does your model show?

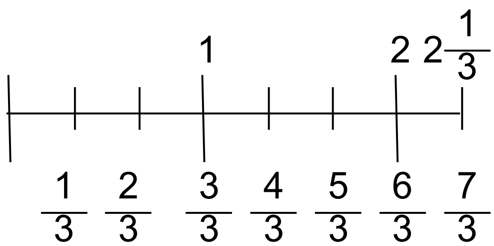
⅓+⅓+⅓+⅓+⅓+⅓+⅓=7/3

1 + 1 + ⅓ =2 ⅓

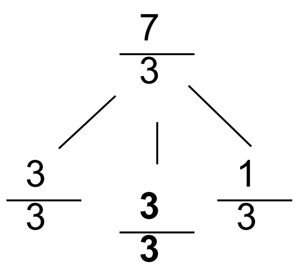


Student #2: Number line

Draws 7/3, then counts 1,2,3 (1 whole) 1,2,3 (2 wholes),
Writes the improper fractions below
(Hopefully)

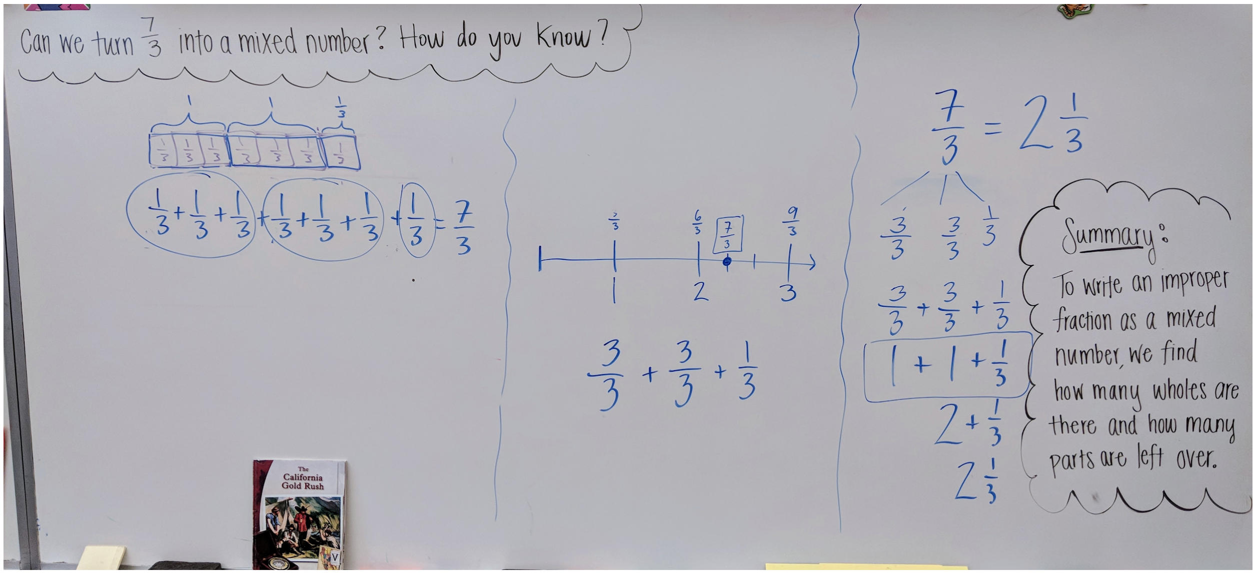


Student #3: Shows decomposition



DRAMA here! How did you know 3/3 and 3/3? Do we see that in the other models?
Answer: 7/3 = 2 ⅓
Where is the 2 ⅓ in each model?

Summary: To write an improper fraction as a mixed number, we find how many wholes are there and how many parts are left over



In our board work, we are working to use a consistent, measurable sized one whole.

14. Reflection

Final reflection (within a week of the lesson)

1. What we learned about the mathematical topic
    a. Fraction strips assist in having a consistent one whole and a tool for measurement. In becoming more independent, we would want students to connect concepts of division with fractions in their use of graph paper: 12 units = one whole would be a useful length of a fraction bar.
    b. Sarah-fractions are numbers, we need to intro fractions greater than one and use linear models and number lines
    c. Hanna- keeping a consistent whole on the board, having permission to focus on the magic of fractions
    d. Julie- the size of the whole is important and it helps so much to keep this in mind on the board
    e. Billy- grounding things in a unit fraction- it is part of a number system, but not confused with the number 2 or 3
    f. Jana- teaching and learning are interconnected, we as teachers need to learn in meaningful ways so that our kids can benefit, this is the beauty of lesson study, thinking about progression
    g. Kari- i am way more excited about teaching fractions, I get what is so hard about them and what is so amazing about fractions as numbers, equivalent fractions are a crazy idea, building stronger foundations leads to MUCH stronger lessons after
2. What we learned about our research question and theory of action
    a. From Hanna:
        i. Wed (before lesson) was using manipulatives and making mixed numbers using number line and fraction strips

ii. Thursday: question from Japanese unit: "how much is 4 1/3s?" and kids made discoveries about numerator being greater than denominator, improper fractions
iii. Didn't anticipate such high comfort level on day of public lesson
iv. How does existing unit plan match our recommendations moving forward?
1. better to see 4/3 on number line first, seeing it as a number of unit fractions. Then students can understand improper fraction as more unit fractions
2. When decimals are taught before improper fractions, mixed numbers would make sense before improper fractions

b. What is the usefulness of each of the forms of fractions (Tad's feedback)
i. We are all fascinated by Tad's observations about how improper fractions tell you how many units and mixed number tells relative size/quanitiy

c. Might be interesting to have class discussion toward end of lesson about the different fractional representations

d. How does pacing of this compare to Eureka pacing?
i. we often feel in crisis about getting through everything. Want to know how this aligns with Eureka pacing
ii. Our public lesson was actually Day 1 of Eureka
iii. We need to ask where we support students--deep conceptual through discovery vs. procedural fluency

e. Impossible to jump into

f. This will all help students add unlike denominators. In 5th, we don't necessarily teach unit fractions, equivalence, etc… but our students haven't always come to us with these understandings

g. We gave excellent pre-assessment, paid attention to the results. Forced us to stop and pay attention to who our studetns are.

h. Fractions: lots of new content, didn't need to rely on old knowledge, allows us to go faster

i. "Go slow to go fast"--hard around SBAC time (may need to direct instruct to get through it all)
i. Word of caution around "coverage" as SBAC approaches. There is compelling research to say it's not necessary to cover it all

j. Debate piece: how do we create argument in class?
i. class vs. teacher
ii. whole class misconception
iii. lesson planning: "can you prove it" vs. "can you show me?"
iv. Build expectation that students will talk with each other during every lesson

3. Implications for our next lesson study cycle -
a. Do the document sooner
b. Will there be another cycle this year? Yes, K-1

c. Identify a pre-assessment (hopefully without creating), and make it a priority, make it a requirement for all Lesson Study cycles moving forward
d. Great having cross-grade level opportunities with similar content, also focusing on unit across years. Wish we'd had 3rd grade team
e. Should focus on Coherence. Lots of cross-over in Progressions
f. What is the relationship between foundational understandings at each grade level
g. What kind of questioning should happen once board work is up? Good practice to ask questions of multiple kids, not just the presenter (what was XX thinking? Who can explain the strategy…)
h. Van de Walle early, then progressions, Graham Fletcher videos

4. Implications for our school
    a. Make sure there is a quality pre-assessment
    b. SH: doesn't need to be a Lesson Study cycle focused on math in order to focus on TRU Dimension. Not fair to expect teachers to have to learn so much about content at the same time as focusing on the five dimension of mathematical powerful classrooms -- not during lesson study cycle
        i. five dimension of mathematical powerful classrooms could be focused on for PLC or observations
    c. Conversations in math -- there is a shift now that the format of the lesson has changed (TTP format)
        i. This doesn't necessarily needs to be the focus, but it does need to go hand in hand
    d. "My friend's thinking…" from Dr. T was mind blowing. Who cares about the summary sometimes, have the learning be about what you learned from a friend! Be sure to allow time to reflect on what a friend shared, pause and give that importance, make it meaningful. Make sure we provide opportunity for students to revise their thinking
    e. Before writing summary on board, we want students to reflect, revise, write in notes, etc…
    f. Summary is valuable for some.
    g. Not every lesson ends in a pretty summary!
    h. Need to be thoughtful and ensure summary actually resonates for kids
    i. Using student reflections as a lesson!
    j. Bring kids back to their journals to re-read their own reflections
    k. what is the role of misconceptions?
5. Implications for our district
    a. Congrats!
    b. Committed to lesson study
        i. Has evolved over time and excited about commitment at AWE to LS professional development
        ii. powerful value of PD to both students and teachers

iii. the nature of support is to eliminate the need for support over time. AWE is headed in that direction through school-wide LS
iv. Lesson Study keeps PD as close as possible to the students--ideal

c. Hoping funding situations will change so we can continue to directly support
   i. District will continue with partnerships, funding opportunities, continued support
d. Looking forward to AWE team sharing experiences with other OUSD schools both in recruitment and with new schools to lesson study cycles and at Saturday Sessions

6. Next steps
   a. download of what was discovered--what we learned about the progression of fractions
   b. How do we share our team learning with the rest of the school / community?

7. Did we learn what we set out to learn?
   a. Diving into fraction models -- when do we use what? (Julie)
   b. How to identify denominator, shift to thinking about it's size, teaching implications, progression, impact going forward (Sara)
   c. How do I set my students up for success? Be intentional about teaching fractions, best ways for teaching fraction concepts (fractions in 4th turns kids away), transformative math experience for kids (Hanna)
   d. Which models are most helpful? What are limitations, what are common misconceptions? (Kari)
   e. Differentiating, making TTP work, see a struggling student present an idea (Billy)
   f. Share learning. How do we identify partial understandings and identify where students are along the progression and address the unfinished learning (Robin)
   g. Mock-up with Dr. T (Jana)
      i. some of the best learning happened on Wed. before with Dr. T. Would have been a disappointing take away from the public lesson, but was powerful on Wed. (e.g., consistent size of whole)

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Dr. T's Comments:

- We have been impatient! This is a brand new concept! They won't fully understand in one day. One example is not sufficient to allow generalization. Wait until the next day: "let's see if it works for other numbers!" "Today we were able to have two names for the same place on the number line!" Or "We were able to show the improper fraction as a mixed number."

- Mathematics of equivelence created conversation "We don't have to put the label because we already put one! No, we can also call it 3/3!" That is a big learning.
- Celebrate the amount of independent work without any teacher instruction! Great job with the timing, including so much ind. work time
- For kids to solve on their own builds their identity as mathematicians
- Reflections that show "I want to learn…." are fantastic, it's great to base the next days work on this.
- Almost all kids were able to execute an answer on their own. Hanna didn't say "use __ model," and she didn't need to. 9 kids used number line, 15 tape diagram, only 5 kids got wrong or no answer.
- Recording "My friend's thinking" is very helpful to allow kids an opportunity to correct thinking. Mistakes are gifts if kids reflect and learn from them. Make time at the end for reflect and revise and record friend's thinking BEFORE summary creation.
- Make sure that students who present are working to teach their classmates, not their teacher. T can even move to the back of the class to ensure that kids face classmates
- Research is critical to the TTP process. The test scores tell us where they are weak but not how to fix the problem! Data plus research along into best practice is the best way to improve.
- Units cannot be "perfect" fit for all schools! Lessons have to be custom designed for each group of students. This is critical for all units of instruction, not just

- 10 min whole class
- 7 min ind
- 9 min partner
- 12 min whole class
- 1 min turn and talk
- 8 min whole class
- 7 min independent

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