Unit and Lesson Plan for Grade 5

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1. Title of the Unit: Ways of counting and mathematical expressions

2. Brief description of the Unit

This unit is designed for students to be able to represent, using mathematical expressions, various ways of counting; to understand what each number in the mathematical expression is representing; and to use those mathematical expressions to solve problems

3. Goals of the Unit:

4.OA. 5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

4. Relationship of the Unit to the Standards

3.OA. Solve problems involving the four operations, and identify and explain patterns in arithmetic.
4.OA. Use the four operations with whole numbers to solve problems.
5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

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5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
6.EE.9 Represent and analyze quantitative relationships between dependent and independent variables.

5. Background and Rationale

This unit is designed for Grade 5 students to understand the usefulness of mathematical expressions through solving a series of problems by themselves and comparing their solutions with several different approaches by their peers.

Since kindergarten, students have been using mathematical expressions to express quantitative relationships in story problems with everyday contexts. From their experience they began to understand the usefulness of mathematical expression for determining what operation can be used to solve problems.

Building upon their prior learning, the students will experience communicating their own reasoning to others by expressing their own ways of counting in mathematical expressions.

At the same time, other students will have opportunities to infer their peers' ways of thinking by interpreting their mathematical expressions. Throughout, students will be expected to justify the reasonableness of their own and others' counting strategies and mathematical expressions. These experiences are expected to foster their skill for developing viable arguments and critiquing the reasoning of others (Standard for Mathematical Practice 3), while the process of writing and interpreting mathematical expressions helps students develop the ability to reason quantitatively and abstractly (SMP 2).

Students will also experience the value of mathematical expressions for determining the number of objects when counting one by one is too difficult, by recognizing the generalizability of the mathematical expressions they write to express their way of counting. Through these experiences students are expected to find the usefulness of mathematical expressions as models to express mathematical situations. As a result it is expected that the students will grow in their ability to use mathematical expressions to analyze relationships mathematically and draw conclusions (SMP 4). This creates the foundation for student to eventually be able to represent, using variables, two quantities that change in relationship to one another.

6. About the Unit and the Lesson

The unit is designed based on the problems from the Japanese curriculum, *Mathematics International* Grade 4 and Grade 5.

In the first lesson, students will explore a task by using a variety of ways of counting. Specifically, the students will discuss how to count the number of dots in the diagram shown as Figure 1, (Mathematics International, Grade 4 p. B13).

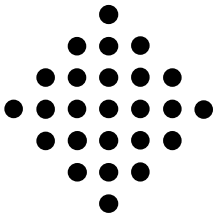


Figure 1.

First, the students will work independently to come up with creative ways of counting the total number of dots. Then each student will express his/her own ways of counting in a mathematical expression.

Second, through whole class discussion, the students will be given the opportunity to share their own mathematical expressions with the class, and the class will try to infer the method of counting by interpreting students' mathematical expressions. After the discussion the class will summarize ideas for coming up with creative ways of counting the dots arranged in such diagram.

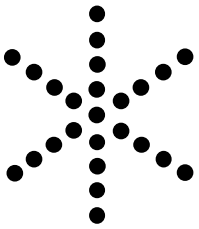


Figure 2.

In the second lesson, students will be given the opportunity to use what they learned from the first lesson. The diagram shown as Figure 2 will be given and students will discuss effective ways to organize the dots into equal groups. In addition to activities similar to those in lesson 1, students will be asked how many dots there will be if the dots in each arm of the star shape increase. This question is designed for students to explore the generalizability of mathematical expressions.

The dots in the diagrams from lessons 1 and 2 can be easily counted one-by-one, which makes it easy to verify the results obtained from mathematical expressions.

In the third lesson, based on what students learned from lessons 1 and 2 about how to use mathematical expressions to show their ways of counting dots, students are given the opportunity to find the number of dots without counting one-by-one. The diagram shown as Figure 3 will be given to the students and they will be asked to find out the total number of dots. To expand the learning,

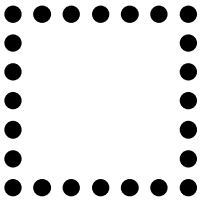


Figure 3.

students will be asked to find out the total number of the dots when each side of the diagram becomes 10 dots. Although students will be encouraged to find out the total number without drawing the diagram, doing so is still possible for any student who really needs it. This task provides a transition for students to see the possibility of finding the total number of dots in the absence of an actual diagram, by using mathematical expressions. The students are expected to recognize on their own that the numbers in their mathematical expressions may be generalized.

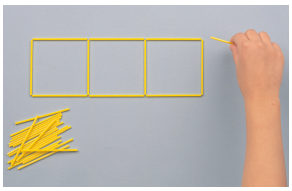


Figure 4.
Reprint of MI
Grade 5 p. A103

The fourth lesson is designed for students to apply all their learning from the previous three problem solving lessons to a different situation. The stick problem, as shown as Figure 4, will be presented: students will be asked to determine the number of sticks required to make a row of 30 adjacent squares. Students will explore a task in which actual counting is not feasible so that they can utilize what they experienced in previous lessons about the generalizability of mathematical expressions. Although the arrangement of sticks will be shown as in Figure 4, the actual number of sticks they are going to find is large enough that they cannot draw a diagram and count the number of sticks one-by-one. Students are expected to come up with their own mathematical expressions to find the total number of sticks needed. In the whole class discussion, students will examine various mathematical expressions to see if they are reasonable.

Each of the four problem solving lessons described above will foster students' reasoning by challenging them to come up ways to find the number of objects in patterns, to express their ways of thinking in mathematical expressions, and to justify whether a mathematical expression is reasonable. At the same time, students will have opportunities to discuss the rules of arithmetic calculation (such as the use of parentheses and the order of operations).

7. Flow of the Unit

Lesson	**Learning objective(s)**	
1	Let's think about ways to count the number of dots. Students will try to express their ways of counting dots in mathematical expressions and also infer other students' ways of counting from their mathematical expressions.	60 min
2	Use mathematical expressions to show your ways of counting. Students will be able to express their ways of counting dots, using mathematical expressions, and will also infer other students' ways of counting from their mathematical expressions.	60 min
3	Find the number of dots without counting one-by-one Students will be able use mathematical expressions to express the number of dots and use the expressions to find the total number of dots without counting one-by-one.	60 min
4	How many sticks are there all together? Students will express their ways of counting sticks in mathematical expressions and use their mathematical expressions to determine the number of sticks even when they cannot actually count the number of sticks.	60 min
5	Summary and Exercises	60 min

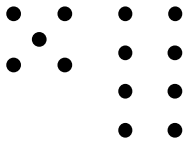
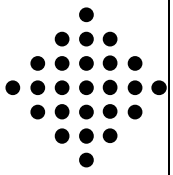
8. Research Lesson Plans

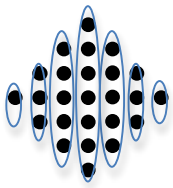
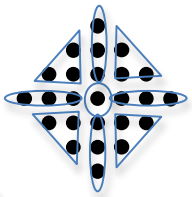
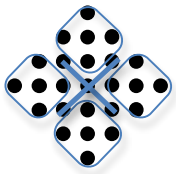
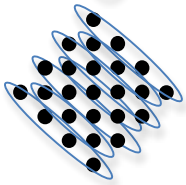
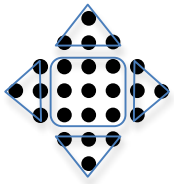
Day 1

(1) Objectives

- Students will express their ways of counting dots in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Examine mathematical expressions using their prior learning of the order of operations and use of parentheses.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Understand how to use mathematical expressions to express ways of counting, in simple cases. Using the two diagrams shown on the right, help students see how mathematical expressions can be used to show ways of counting the number of dots. 4+1=5 2x4=8 4x2=8 Remind students that 2x4 and 4x2 are different ways to look at the diagram. 	By briefly showing the diagrams one at a time, encourage students to share their ways of counting the number of dots in the diagrams. Remind students that multiplication is (number of groups)x(number in each group)	Students understand what it means to use a mathematical expression to show their way of counting and are ready to solve the problem
2. Posing the Problem Show the diagram shown on the right and ask the following question. Think about ways to count the number of dots in the picture shown on the right. For each way of counting, write a math sentence (mathematical expression) that describes the way of counting. 	If students seem to not understand the task, share a few of the students' attempts at expressing their ways of counting in mathematical expressions as examples.	Students understand that the task is to express ways of counting using mathematical expressions.

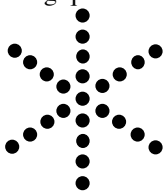
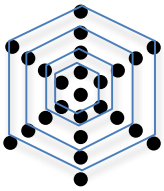
3. Anticipated Student Responses 1) Decompose 25 in arbitrary ways and show them in mathematical expressions. e.g. 10+10+5=25, 12x2+1=25 2) Making groups of dots in the diagram. e.g. 1+3+5+7+5+3+1=25, or 2x(1+3+5)+7=25  3) Making equal groups to use multiplication e.g. 8x3+1=25  5x5=25  3x3+4x4=25  	Let each student write their ways of counting and mathematical expressions in their notes. Use a seating chart to note each student's way(s) of counting and mathematical expression(s), in order to prepare for organizing the whole class discussion.	Each student comes up with at least one mathematical expression to express how to count the number of dots.
4. Comparing and Discussing For each mathematical expression, 1) Ask one of the students who came up with the mathematical expression to show it to the class, 2) Let other students infer how the student counted the dots by interpreting the mathematical expression, and 3) Let the student who came up the mathematical expression justify if the other student's inference is correct. Repeat the above to understand the variety of ways to count the number of dots by making equal groups.	By providing opportunity to infer other students' ways of counting, help students see that mathematical expressions can be a communication tool for understand others' thinking. Help them recall their prior learning such as the meanings of operations and the order of operations and examine if they can express their ways of counting correctly.	Students can use their prior learning such as the meanings of the operations and the order of operations to express their ways of counting correctly in mathematical expressions. Students understand that they can infer a way of counting from an expression.
5. Summing up Help each student highlight the learning from the class and record it in their notes. • Mathematical expressions can be used to show ways of counting the number of the dots. • Making equal groups will help you use multiplication to find the total number of dots easily.		Students summarize their learning and record it in their notes.

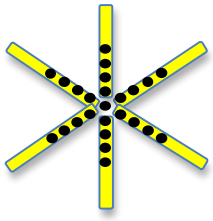
Day 2

(1) Objectives

- Students will use learning from Day 1 to become comfortable expressing their ways of counting dots in mathematical expressions and inferring other students' ways of counting from their mathematical expressions.
- Use their prior learning of the order of operations and use of parentheses to express and interpret mathematical expressions correctly.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Let a few students read their journal reflections from Day 1 and help the class recall what they learned on Day 1.	Select a few exemplary journal reflections from Day 1 before class.	Students are ready for the new problem.
2. Posing the Problem Show the diagram on the right and ask the following question. Think about ways to count the number of dots in the picture shown on the right. For each way of counting, write a mathematical expression that describes that method of counting. 	If students seem to not understand the task, share a few of the students' attempts at expressing their ways of counting in mathematical expressions as examples.	Students understand that the task is to express ways of counting with mathematical expressions.
3. Anticipated Student Responses 1) Making groups of dots in the diagram. e.g. 9+4+4+4+4=25  2) Making equal groups to use multiplications e.g. 6x4+1=25  1+4x6=25  3) Decompose 25 in arbitrary ways and show them in mathematical expressions. e.g. 10+10+5=25, 12x2+1=25	Let each student write their ways of counting and mathematical expressions in their notes. Use a seating chart to note each student's way of counting and mathematical expression, to prepare for organizing the whole class discussion.	Each student comes up with at least one mathematical expression to express how to count the number of dots.
4. Comparing and Discussing For each mathematical expression, 1) Ask one of the students who came up with a mathematical expression to show it to the class, 2) Let other students infer how the student counted the dots by interpreting the mathematical expression, and 3) Let the student who came up with the mathematical	By providing an opportunity to infer other students' ways of counting, help students see that a mathematical expression can communicate a way of	Students can use their prior learning such as the meanings of operations and the order of operations

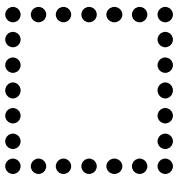
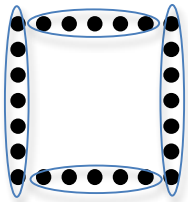
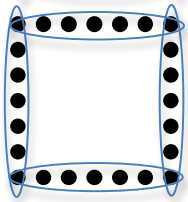
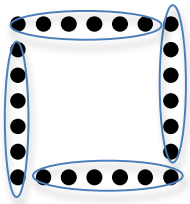
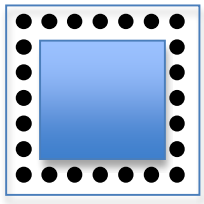
expression justify if the other student's inference is correct. Repeat the above so students understand a variety of ways to count the number of dots by making equal groups.	thinking. Help them recall their prior learning, such as the meanings of operations and the order of operations to examine if they can express their ways of counting correctly.	to express correctly their ways of counting.
5. Expanding the learning Using a mathematical expression developed when there were 4 dots on each arm of the star shape, determine the total numbers of dots when there are 10 dots on each arm of the shape.  Ask a few students if they can use their mathematical expressions and how.	Encourage students to find the total number of the dots without drawing all the dots. Help students understand that arbitrary expressions for 25 are not useful.	Students try to use a mathematical expression to find the total number of dots.
5. Summing up Help each student identify the learning from the class and record it in their notes. • Mathematical expressions can be used to show ways of counting the number of the dots. • Mathematical expressions may be used to find the number of dots even without seeing the actual diagram.		Each student summarizes their learning and records it in their notes.

Day 3

(1) Objectives

- Students will express their ways of counting dots in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will use the mathematical expressions they developed while counting the dots for the case of 7 dots on each side of the square to determine the total number of dots when the number of dots on each side changes.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	Points of Evaluation
1. Introduction Let a few students read their journal reflections from Day 2 and help the class to recall what they learned on Day 2.	Select a few exemplary journal reflections from Day 1 before the class.	Students are ready for the new problem.
2. Posing the Problem Showing the diagram on the right, ask the following question. Think about ways to count the number of dots in the picture shown on the right. For each way of counting, write a mathematical expression that describes that method of counting. 	Help students see the characteristics of the shape. There are 7 dots in each side of the square.	Students understand that the task is to express ways of counting, using mathematical expressions. Students understand 4x7 is not correct.
3. Anticipated Student Responses 1) Making groups of dots in the diagram. e.g. 7+5+7+5=24, 2x7+2x5=24, or 2x(7+5)  2) Making equal groups to use multiplications. e.g. 4x7-4=24  4x5+4=24 or 4x(7-2)+4=24 4x6=24 or 4x(7-1)=24  3) Apply the idea of finding the area of a square. 7x7-5x5=24 	Let each student write their ways of counting and mathematical expressions in their notes. Use a seating chart to note each student's way(s) of counting and mathematical expression(s), to prepare for organizing the whole class discussion.	Each student comes up with at least one mathematical expression to express how to count the number of dots.
4. Comparing and Discussing For each mathematical expression,	By providing an	If students be able

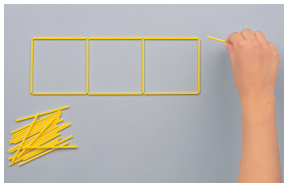
1) Ask one of the students who come up the mathematical expression to show to the class, 2) Let other students infer how the student counted the number of dots, by interpreting the mathematical expression, and 3) Let the student who came up with the mathematical expression justify if the other student's inference is correct. Repeat the above to understand a variety of ways to count the number of dots by making equal groups. Discuss what each number in the mathematical expression is representing. e.g.) • 4 in the mathematical expressions represents the number of sides of the shape because it is a square. • 6 is 1 taken away from 7 and 5 is 2 taken away from 7.	opportunity to infer other students' way of counting, help students see that mathematical expressions can be a communication tool for understanding others' thinking. Help students recall their prior learning such as the meaning of operations and order of operations and examine if they can express their ways of counting correctly.	to use their prior learning such as the meanings of operations and the order of operations to express their ways of counting correctly in mathematical expressions.
5. Expanding the learning Using a mathematical expression developed when there were 7 dots on each side of the square, determine the total number of dots when there are 10 dots on each side of the square. Ask a few students if they can find the total number without drawing the shape.	Encourage students to see which number or numbers in the mathematical expression would be different.	Students try to use a mathematical expression to find the total number of the dots.
5. Summing up Help each student highlight the learning from the class and record it in their notes. • Mathematical expressions to find the number of dots in the first problem can be used to find the number of dots in the second problem even without seeing the actual diagram.		Each student summarizes their learning and records it in their notes.

Day 4

(1) Objectives

- Students will express their ways of counting sticks in mathematical expressions and infer other students' ways of counting from their mathematical expressions.
- Students will determine the number of sticks when the number of squares is 30, using mathematical expressions they developed while counting cases of small number of squares.
- Students will confirm that they can use their mathematical expressions to determine the number of sticks even when they cannot actually count the number of sticks because the expressions are based on their ways of counting.

(2) Flow of the lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	**Teacher's Support**	**Points of Evaluation**
1. Introduction Let a few students read their journal reflections from Day 3 and help the class to recall what they learned from Day 3.	Select a few exemplary journal reflections from Day 3 before class.	If students are ready for the new problem.
2. Posing the Problem Show the diagram on the right and ask the following question. Using sticks of the same length, we will make squares side by side as shown on the right. To make 30 squares, how many sticks are needed? 	If students do not understand the situation, guide them to see how the number of sticks increases as the number of squares increases. Use animation to show how the number of squares will increase.	Students understand the situation.
3. Anticipated Student Responses By arranging sticks or drawing a diagram, student thinks about ways to count the sticks more easily. (a) Notices that the number of sticks increases by 3 when 1 square is added and writes a mathematical expression. • $4 + 29 \times 3 = 91$ • $4 + (30-1) \times 3 = 91$ (b) Each square has 4 sides, so calculate 30×4. Then, since the sides between two adjacent squares are double counted, subtract the number of overlapping sides, 29 (or $30 - 1$), from the product. • $30 \times 4 - 29 = 91$ • $30 \times 4 - (30 - 1) = 91$ (c) Since there are 30 sticks at the top and 30 sticks at the bottom, 2×30. The number of vertical sticks is $30 + 1$. • $2 \times 30 + 31 = 91$ • $2 \times 30 + (30 + 1) = 91$. By using a drawing of a diagram as the number of squares is increased, explore the number of sticks in relationship to the number of squares. (d) Notices that the number of sticks increases by 3 when 1 square is added and writes a mathematical expression. • $4 + 29 \times 3 = 91$ • $4 + (30-1) \times 3 = 91$ (b) Notices that the number of sticks is increasing by 3 but cannot write a mathematical expression. Misconception e. g. $30x4=120$	If students cannot think about ways of counting without drawing, encourage them to use an example case, such as the case with 5 squares, to come up with a way to count the total number of sticks, and express it in a mathematical expression. Then use it to think about which number in the mathematical expression will be different if the number of squares is 30. Use seating chart to note each student's way(s) of counting and mathematical expression(s) to prepare for organizing the whole class discussion.	Each student comes up with a mathematical expression to express how to count the number of sticks.

4. Comparing and Discussing By using a drawing of the diagram as an example (e.g. diagram with 5 squares), ask students to share how they counted the number of sticks. • Try to figure out ways of counting represented by mathematical expressions. Let students share their ideas for finding the number of sticks when there are 30 squares. • Using the strategies discussed when there were 5 squares, students will examine how many sticks will be used when there are 30 squares.	For the students who do not understand a way of counting 30 squares, the whole class discussion begins by letting them develop a mathematical expression by looking at the case of 5 squares.	Do students understand ways of counting the number of sticks using the example?
5. Expanding the learning Using a mathematical expression developed when there were 5 squares, determine the total number of sticks when there are 30 squares.	Encourage students to see which number in the mathematical expression would be different.	Students try to use the mathematical expression to find the total number of sticks.
5. Summing up Help each student highlight the learning from the class and record it in their notes. • The same mathematical expressions for finding the number of sticks may be used to find the number of sticks for various numbers of squares even without seeing the actual diagram.		Each student summarizes their learning and records it in their notes.

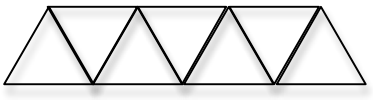
Day 5

(1) Objectives

- Thought solving the practice problems the students will become familiar with expressing mathematical situations using mathematical expressions.
- Students will be able to solve problems by recognizing the pattern exists in the situations, express them using mathematical expression, and solve the problem by using the mathematical expressions.

Practice Problems

Using sticks of the same length, we will make triangles side by side as shown on the right.
To make 30 triangles, how many sticks are needed?



Using sticks of the same length, we will make triangles side by side as shown on the right.
How many triangles can you can make if you have 46 sticks?