## What's Hard About Fraction Number Sense?

Type of Understanding or Knowledge	Example of Student Difficulty or Understanding	Tasks and Experiences That Build This Understanding (For your notes)
<ul> <li>A Fraction is a Number</li> <li>A fraction represents an amount, not just pieces (such as 2 of 3 pieces of a pizza) or a situation (such as 2 of 3 shirts are red).</li> </ul>	<ul> <li>When asked to put the fraction <sup>2</sup>/<sub>3</sub> on a number line, a student said "you can't put it on a number line, because it's two pieces out of three pieces, it's not a number." (Or "<sup>2</sup>/<sub>3</sub> is not a number, it's two numbers.")<sup>6</sup></li> </ul>	
<ul> <li>Partitioning Fractions</li> <li>A whole can be divided into smaller and smaller equal parts.</li> <li>The same quantity can be represented by different fractions.</li> </ul>	<ul> <li>Does not divide a whole into equal parts.</li> <li>Difficulty seeing that <sup>1</sup>/<sub>2</sub> is equal to <sup>2</sup>/<sub>4</sub>, <sup>3</sup>/<sub>6</sub>, <sup>4</sup>/<sub>8</sub>, <sup>5</sup>/<sub>10</sub>and so on.</li> </ul>	
<ul> <li>The Meaning of the Denominator</li> <li>Different units (such as <sup>1</sup>/<sub>3</sub> and <sup>1</sup>/<sub>5</sub>) are different sizes.</li> <li>The more units a whole is partitioned into, the smaller each one is.</li> <li><sup>1</sup>/<sub>n</sub> fits exactly <i>n</i> times into the whole.</li> </ul>	<ul> <li>Students add <sup>1</sup>/<sub>3</sub> + <sup>1</sup>/<sub>5</sub> and get <sup>2</sup>/<sub>8</sub>, without realizing they are adding two different things (thirds and fifths), sort of like adding apples and hammers.</li> <li>Students may think "<sup>1</sup>/<sub>5</sub> is bigger than <sup>1</sup>/<sub>4</sub> because 5 is bigger than 4."</li> <li>Difficulty seeing that <sup>1</sup>/<sub>3</sub> fits in the whole 3 times, <sup>1</sup>/<sub>4</sub> fits in the whole 4 times. Difficulty seeing that <sup>3</sup>/<sub>3</sub>, <sup>4</sup>/<sub>4</sub> etc. equal 1.</li> </ul>	
<ul> <li>Knowing What is the Whole</li> <li>Constructing the whole when given a fractional part.</li> <li>Keeping track of the whole.</li> </ul>	<ul> <li>Difficulty making the whole when you give them a fractional part, e.g.: "This paper is <sup>2</sup>/<sub>3</sub>; show me the whole."</li> <li>Sees that the magnitude of a fraction depends on the magnitude of the whole (e.g., half of a small cookie is not the same as half of a large cookie).</li> </ul>	

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	• Confusion about whether the two drawings below together represent $\frac{3}{8}$ of a pie or $\frac{3}{16}$ of a pie.	
<ul> <li>Fraction Size</li> <li>Understands that fraction size is determined by the (multiplicative) relationship between numerator and denominator - not just by the numerator, not just by the denominator, and not by the <i>difference</i> between numerator and denominator.</li> <li>Sees non-unit fraction as an accumulation of unit fractions. (A unit fraction has a numerator of 1; a non-unit fraction has a numerator other than 1.)</li> </ul>	<ul> <li>May think <sup>4</sup>/<sub>9</sub> is bigger than <sup>3</sup>/<sub>4</sub> because 4 is bigger than 3 (comparing numerators), or <sup>4</sup>/<sub>9</sub> is bigger than <sup>3</sup>/<sub>4</sub> because 9 is bigger than 4 (comparing denominators), or <sup>3</sup>/<sub>5</sub> is the same size as <sup>5</sup>/<sub>7</sub> because the difference between the top and the bottom in both fractions is 2.</li> <li>Sees that equivalent fractions have the same multiplicative relationship between numerator and denominator. In <sup>2</sup>/<sub>4</sub>, <sup>4</sup>/<sub>8</sub>, <sup>3</sup>/<sub>6</sub>, etc. denominator is two times numerator.</li> <li>Sees <sup>5</sup>/<sub>8</sub> is made up of 5 eighths or 5 times <sup>1</sup>/<sub>8</sub>, that <sup>9</sup>/<sub>8</sub> is made up of 9 eighths or 9 times <sup>1</sup>/<sub>8</sub>, etc.</li> </ul>	
<ul> <li>Fractions Can Represent</li> <li>Quantities Greater than One</li> <li>May be difficult for students who have a strong image of a fraction as a <i>piece</i> of something.</li> </ul>	• "You can't have $\frac{6}{5}$ , because there's only $\frac{5}{5}$ in a whole."	