Challenge in Understanding	Example of Student Difficulty	How Might a Linear Measurement Context Help?
A Fraction is a Number • A fraction represents an amount, not just pieces (such as 2 of 3 pieces of a pizza) or a situation (such as 2 of 3 shirts are red).	• When asked to put the fraction $^2/_3$ on a number line, a student said "you can't put it on a number line, because it's two pieces out of three pieces, it's not a number." Or " $^2/_3$ is not a number, it's two numbers." [*]	• Linear measurement may help students shift from thinking "how many pieces" (counting) to thinking "how much" or "how long" (relative size). • Students can partition a whole themselves and check whether the parts are equal; it is relatively easy to compare whether 2 lengths are equal, and to notice the relative size of part/whole.
Fractions Can Be Greater than One • May be difficult for students who have a strong image of a fraction as a *piece of something*.	• "You can't have $^6/_5$ because there's only $^5/_5$ in a whole."	• When students measure an object that is longer than 1 foot (meter, etc.), it may be relatively easy to visualize something as a whole plus an additional fractional part and understand the meaning of fractions greater than 1.
Fractions Can Be Partitioned • A whole can be divided into smaller and smaller equal parts. • The same fractional quantity can be represented by different fractions.	• Difficulty seeing how to divide a whole into *equal* parts. • Difficulty seeing that $^1/_2$ is equal to $^2/_4$, $^3/_6$, $^4/_8$, $^5/_{10}$ and so on.	• A number line (or ruler) may make it easy to see that the same point can be described by different fractions.
What the Denominator Means • Different units (such as $^1/_3$ and $^1/_5$) are different sizes. • The more units a whole is partitioned into the smaller each one is.	• Students add $^1/_3 + ^1/_5$ and get $^2/_8$, without realizing they are adding two different things (thirds and fifths) sort of like adding apples and hammers.	• Compared to fractional parts of area (which can be rearranged in many ways), length may provide a clear image of what is $^1/_3m$, $^1/_2m$, $^{10}/_{11}m$, etc., and their relative size. • Linear measurement may help provide a strong image that the unit that fits in 3 times is longer than the

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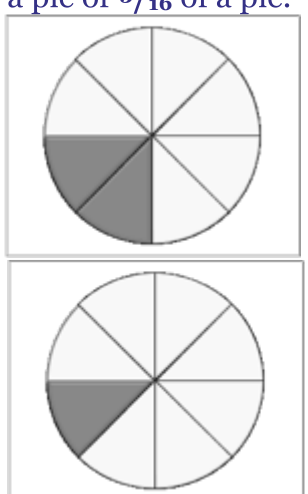
• $^{1}/_{n}$ fits exactly *n* times into the whole.	• Students may think "$^{1}/_{5}$ is bigger than $^{1}/_{4}$ because 5 is bigger than 4." • Difficulty seeing that $^{1}/_{3}$ fits in the whole 3 times, $^{1}/_{4}$ fits in the whole 4 times. Has trouble seeing that $^{3}/_{3}$, $^{4}/_{4}$ etc. equal 1.	one that fits in 4 times, that $^{1}/_{6}$ is half the length of $^{1}/_{3}$, etc.
Knowing What is the Whole • Constructing the whole when given a fractional part. • Keeping track of the whole.	• Difficulty making the whole when you give them a fractional part, e.g.: "This paper is $^{2}/_{3}$; show me the whole." • Sees that the magnitude of a fraction depends on the magnitude of the whole (e.g., half of a small cookie is not the same as half of a large cookie) • Confusion about whether the two drawings below together represent $^{3}/_{8}$ of a pie or $^{3}/_{16}$ of a pie. 	• Using a standard measurement unit may be clearer, more familiar, and more stable than an ad hoc unit (such as pie pieces), making it easier to keep track of the whole.

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Fraction Size • Understands that fraction size is determined by the (multiplicative) relationship between numerator and denominator – not just by the numerator, not just by the denominator, and not by the *difference* between numerator and denominator. • Sees non-unit fraction as an accumulation of unit fractions. [A unit fraction has a numerator of 1; a non-unit fraction has a numerator other than 1.]	• May think $^4/_9$ is bigger than $^3/_4$ because 4 is bigger than 3 (comparing numerators), or $^4/_9$ is bigger than $^3/_4$ because 9 is bigger than 4 (comparing denominators), or $^3/_5$ is the same size as $^5/_7$ because the difference between the top and the bottom in both fractions is 2. • Sees that equivalent fractions have the same multiplicative relationship between numerator and denominator. In $^2/_4$, $^4/_8$, $^3/_6$, etc. denominator is two times numerator. • Sees $^5/_8$ is made up of five $^1/_8$'s or 5 times $^1/_8$, that $^9/_8$ is made up of 9 eighths or 9 times $^1/_8$, etc.	• Length measurement may transfer to the number line more easily than some other models, so that students see the relative size of fractions. A familiar standard measurement unit (a meter, foot, etc.) may make it easy to see $^1/_3$ as a length that goes in 3 times, $^1/_4$ as a length that goes in 4 times, etc. • When students think about a turtle who travels in a straight line $^1/_5$ mile a day for 4 days, they may easily develop an image of $^4/_5$ as four fifths—as $^1/_5$ repeated four times along a number line.

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