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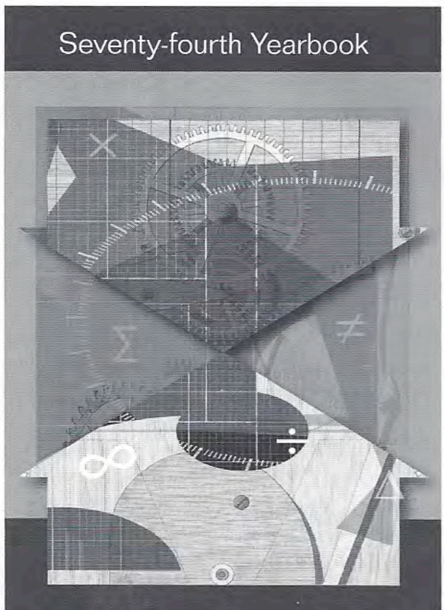

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CHAPTER 18

Learning to Use Student Thinking

Development and Spread of "Re-engagement" Strategies Through Lesson Study

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MAKING STUDENTS' MATHEMATICAL THINKING VISIBLE during classroom instruction is a central challenge of mathematics teaching (Lesh et al. 2000; Boyd et al. 2003; Fisler and Firestone 2006; Ball and Bass 2000). This chapter chronicles how educators in a California lesson study network refined and spread "re-engagement," a teaching strategy designed to make student thinking visible and usable in classroom practice by presenting and discussing students' solutions to tasks. The chapter also examines the characteristics of the regional professional learning community that allowed knowledge about "re-engagement" to develop and spread.

Background on the Silicon Valley Mathematics Initiative and Lesson Study

This case focuses on the San Francisco–Silicon Valley region of California, where the Silicon Valley Mathematics Initiative (SVMI) supported lesson study as one strand of multifaceted work that also included an assessment collaborative and regional network of coaches. In lesson study, a small team of teachers studies the curriculum; plans a "research lesson" that is taught by one team member while the others observe students and collect data; and then discusses the collected data, drawing out the implications for teaching and learning (Wang-Iverson and Yoshida 2005; Lewis 2002; Gorman, Mark, and Nikula 2010). Evidence suggests that lesson study can have a positive effect on teacher and student learning (Perry and Lewis 2010; Lewis, Perry, and Hurd 2009; Fernandez and Yoshida 2004; Lewis et al. 2006; Foster and Poppers 2009). However, little has been written to date about lesson study as a means to support the spread of knowledge across a region or nation. This is a key aspect of lesson study in Japan, where a widely shared knowledge base for teaching has been developed and spread through lesson study (Lewis and Tsuchida 1997).

Overview of the Case

To understand the contours of the case, it is useful to compare lesson study in 2001 and in 2009 in the greater San Francisco-Silicon Valley region. In 2001, a small number of teachers in two districts within this region—Berkeley Unified School District (BUSD) and San Mateo-Foster City School District (SMFCSD)—had independently initiated lesson study groups, but they had not yet collaborated across district lines to observe live instruction. An inspection of research lesson plans from 2001–02 indicates that none of the lesson plans prepared by BUSD and SMFC teachers included presentation and discussion of student work as a way to introduce and consider mathematical ideas. Rather, the lessons were structured around teacher-developed or textbook activities intended to lay the groundwork for "correct" student thinking.

By 2009, teachers in more than twenty districts in the region had initiated lesson study, and many hundreds of teachers had observed and discussed lessons taught by educators from other districts. During every year from 2004–05 through the present, lesson study teams from fourteen to twenty-one districts chose to participate in a regional lesson study network that included public research lessons and exchange lessons in which lesson study teams partnered to observe each other's research lessons. By 2009, research lesson plans routinely anticipated student thinking, described the particular student work to be presented during the lesson, and noted the mathematical points to be drawn out. Teachers in at least seven districts were systematically testing and refining methods to present and discuss student work.

While recording research lessons in SMFCSD and BUSD, we noticed that teachers sometimes credited colleagues in other districts for elements of lesson design. In order to understand how ideas about mathematics teaching had spread across districts, we selected one element, "re-engagement," mentioned in research lessons in SMFCSD and BUSD, and we traced it backwards from 2009 to 2002. We discovered that it had spread across many districts of the San Francisco–Silicon Valley region; we limit the current account to seven lesson study groups in four different parts of the region, in order to use the data (lesson plans and meeting and interview transcripts) available to us. We focus first on the nature of re-engagement, and how teachers developed, refined, and adapted this strategy to solve problems faced in their classrooms. We focus next on the supporting conditions that allowed the development and spread of knowledge about strategies such as re-engagement across the region, arguing that good access to mathematical knowledge and the collaborative, practice-based learning structure provided by lesson study were key factors in knowledge development and spread.

What Is Re-engagement and How Did It Spread?

"Re-engagement" presents strategically selected student work in order to "re-engage" students with a previously explored mathematical idea, elicit their thinking, and help them to examine or re-examine their own thinking and that of other students (Fisher and Keyes 2009; Foster and Poppers 2009). As the examples presented below illustrate, presentation and class discussion of student thinking can help students gain mathematical insights at the same time that teachers gain insights about student thinking. Although teachers in the region did not begin to use the term "re-engagement" until 2006, the roots of this strategy go back to at least 2002, when Linda Fisher (SVMI's director of assessment) participated in a lesson study cycle with mathematics coaches from several districts that were involved in SVMI.

Linda Fisher and the coaches served as lesson study team members for a series of lessons on the area of polygons (Mills College Lesson Study Group 2003) planned and taught by Akihiko Takahashi, who teaches at DePaul University and who served as an elementary teacher in Japan for nineteen years, as well as a teacher educator in Japan. Team members supported this work by collecting data on student thinking during the lessons and by participating in the post-lesson discussions, where the next day's lesson plan was reviewed in light of the data on student thinking. Takahashi taught the series of three consecutive research lessons over a three-day period to a fourth-grade class in SMFCSD. Team members were struck by Takahashi's public use of student work to help students consider key mathematical ideas and misunderstandings. Fisher remarked that Takahashi "slowed down" the lesson and probed student thinking, a process the Japanese call "neriage" (Takahashi 2008). For example, at the beginning of the series of lessons, Takahashi asked students to make a 4-inch by 5-inch rectangle on their geoboards, and to copy their solution onto a worksheet. He posted the solutions shown in figure 18.1, and asked students which was correct. As a student came to the board to show the "four spaces" and "five spaces" along the sides of the 4-inch by 5-inch rectangle, Takahashi marked each space with a curved bracket.

Takahashi asked students to explain why the rectangle on the left is not correct, and followed up with another question:

Takahashi: Why [would] somebody make this shape?

Student: Because they counted the number of pegs, instead of the spaces.

Takahashi: So they counted the number of pegs instead of in between the pegs. Let's see. One, two three four. One, two, three, four, five [*circles one* *"peg" on the drawing with each count, going down the side and then across the bottom of the rectangle*]. OK, so it's very important to count in between to make a shape like this. Do you agree with that?

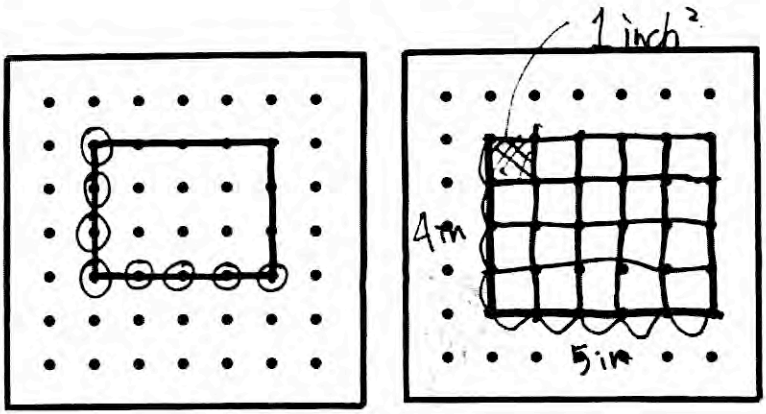

Fig. 18.1. Correct and incorrect examples of 4 × 5 rectangle on board

Likewise, in a discussion designed to connect two different methods for computing the area of a complex figure, Takahashi asked students to compare a strategy that found the area of a 2 × 5 rectangle by adding the area of each row (5 + 5 = 10) to a solution that found the area of the 2 × 5 rectangle using multiplication.

Takahashi: [*To class*] She said this can be five times two. Do you agree with that? Where is five? Where is two? . . .

During the following school year (2003–04) Fisher formed a lesson study team with several network mathematics coaches. The team devoted much of its lesson study time to "pushing on [our own understanding of] the mathematics"—for example, solving proportional reasoning problems, analyzing student work, and reading written resources related to proportional reasoning (including Lo, Watanabe, and Cai [2004]; Singapore Ministry of Education [1994]). Fisher noted that studying the mathematics enabled them to recognize and choose examples of student thinking to discuss with the whole class:

> Playing around and struggling with that mathematics . . . eventually led us to having a really clear idea . . . so when you look at student work you just know . . . You don't have to be doing that thinking on your feet thing that most teachers are doing. It's . . . "Oh yeah, that's an example of this. This is an example of something else." So that as you walked around the room you just see what those examples were that you wanted to pull out . . . But I think it was our own personal struggles that led to that clarity.

In May 2004, three members of the coaches' team co-taught a research lesson centered on the NAEP problem shown in figure 18.2 to a class in Palo Alto Unified School District (PAUSD).

A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars.
How many leaves would they need each day for 12 caterpillars?

Answer: ______________________

Use drawings, words, or numbers to show how you got the answer.

Fig. 18.2. The Caterpillar Problem
(NAEP 1996, Grade 4, reproduced in Kenney, Lindquist, and Heffernan 2002)

Figure 18.3 shows some of the student solutions generated by that lesson. The team decided to include these solutions in the revised lesson plan they prepared for a large public research lesson, after the following discussion:

> And remember when Takahashi did it. . . . For each of his problems, he has … that little page about anticipated student responses? I think it would be really [good] for us to put together … one page of … anticipated responses. So that going around the room you could actually tally up how many people are doing each strategy. And then if a, a new one comes up, you only have to find out that one. . . . I think it's possible now that we have some student work to actually do that.

Another team member added, "If there's one that didn't come up that you wanted, you could just … [present it saying] I saw this in another class."

In May 2004, Fisher taught the public research lesson at the Tech Museum of Innovation, attended by about forty math coaches and teachers from the SVMI network. As she collected students' solutions to the caterpillar problem, Fisher told the students, "A really good friend of mine says that the real lesson starts *after* you solve the problem, when you start to think about how other people have solved the problem. I'd … like to start up here with the way Diego solved the problem." Fisher then supported a classwide discussion of five different solution methods used by the students. When students described their thinking to the class in words like "for every two caterpillars they eat five leaves," she encouraged elaboration by asking "Where's the 2 and where's the 5?" and by asking students to point to these spots in their diagram as well as in other diagrams. The five solution methods were thus used to re-engage students in thinking about the mathematics of this proportional reasoning problem from different perspectives (such as unit rate and multiplicative relationships) and to connect these different mathematical ideas.

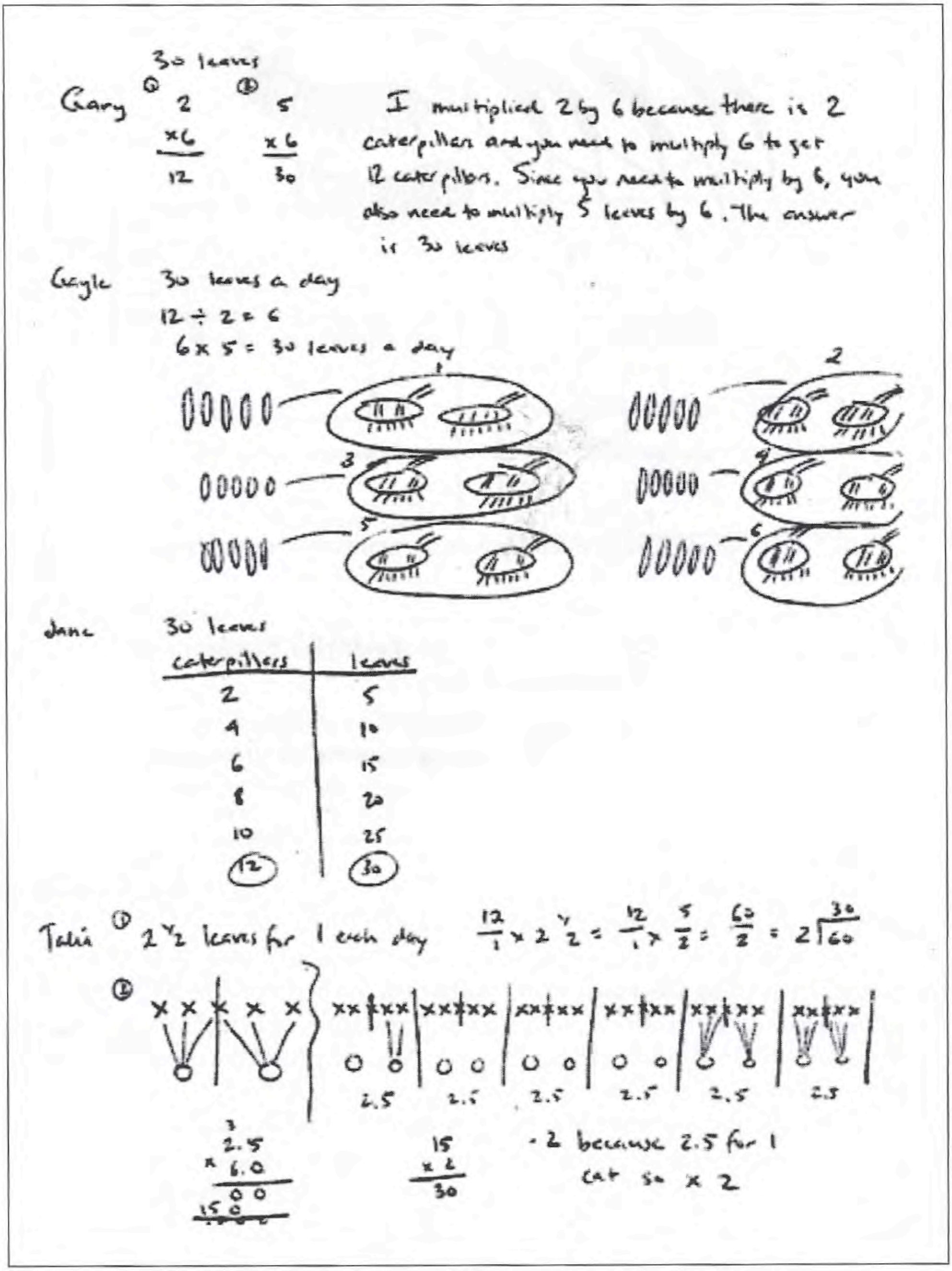

Fig. 18.3. Student work examples from the Caterpillar Problem included in lesson plan for public research lesson

The mathematics teachers from Willard Middle School (BUSD) viewed Fisher's research lesson and were struck by how well their own students (who had been bussed in for the lesson) explained their ideas during the research lesson. Over the next three years, the Willard group began to add to their own research lessons more opportunities for students to explain their thinking and to make connections between different solution strategies. Figure 18.4 shows the board from a research lesson taught at Willard Middle School in January 2007; its left side records student predictions and explanations about the height a ball would bounce from different dropping points. The planning leading up to the lesson included extensive discussion of the mathematics of proportional reasoning, the solution strategies students might come up with (such as unit rate, scale factor and ratio), the importance of each, and the connections among them. The lesson used re-engagement, introducing examples of student thinking from previous lessons. The lesson also asked students to present their ideas to each other in posters and at the board, and the teacher organized student thinking and data as shown. However, as the post-lesson discussion reveals, teachers saw the need for even more thorough planning of how to present and discuss student thinking:

Teacher 2: I think [the teacher] did a great job of getting all the student thinking and trying to make a quick gloss … on that far left-hand side of the board [fig. 18.4], but I think we as a planning group need to think more about … as a next step, how we can display those strategies in a different way, or more clearly … either on poster paper or bigger, somehow differently, so … that kids who didn't use that one … could benefit from it, and see it, and . . . make connections between these strategies. . . . Because … there were a lot of strategies there, and I'm not sure how many groups understood or benefited from the strategies that they didn't use in their … particular group.

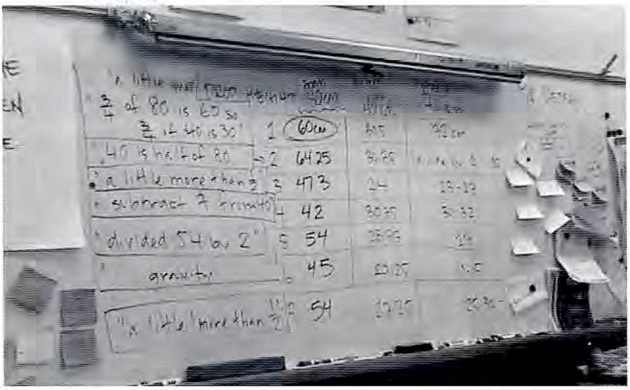

Fig. 18.4. Board from research lesson, Willard School

Meanwhile, Fisher continued to work in lesson study groups and action research groups to examine strategies for choosing, presenting, and discussing student work. These groups tried introducing selected student work (typically from the prior lesson) at the beginning of a lesson so that students could revisit and discuss an important mathematical issue. This strategy, which they referred to as "re-engagement," became a heavy focus of work in the middle school consortium. The Willard lesson study group partnered for an exchange lesson with one of these consortium groups, and they were impressed with the power of re-engagement—for example, the possibility of reintroducing selected student work at the beginning of a lesson so that students could, in the words of Jake Disston:

> Re-engage with these fragile ideas in ways that help them solidify them, and help them weigh the sensibility of them. . . . Instead of just that first pass where they kind of grab something, and it either works or it doesn't work . . . and then you move on.

In a June 2008 research lesson, Willard School teachers used re-engagement to address a particular mathematical concern of their group. Teachers had noticed that some students who used proportional reasoning when problem quantities were easily multiplied reverted to incorrect additive reasoning when confronted with a non-integer scale factor such as 2.5. Figure 18.5 shows the (recopied) student work used to introduce the lesson. The lesson began by asking the class to describe the thinking behind each piece of work, to explore how each type of thinking would influence the shape of the enlargement, and then to apply each type of thinking to a subsequent problem about lemonade solutions, predicting how each type of thinking would affect the "lemoniness" of the solution. In contrast to the bouncing balls lesson, which included many on-the-fly decisions about how to use student thinking, this research lesson planned in advance how the additive and multiplicative thinking from the poster problem would be used to support discussion of the lemonade problem.

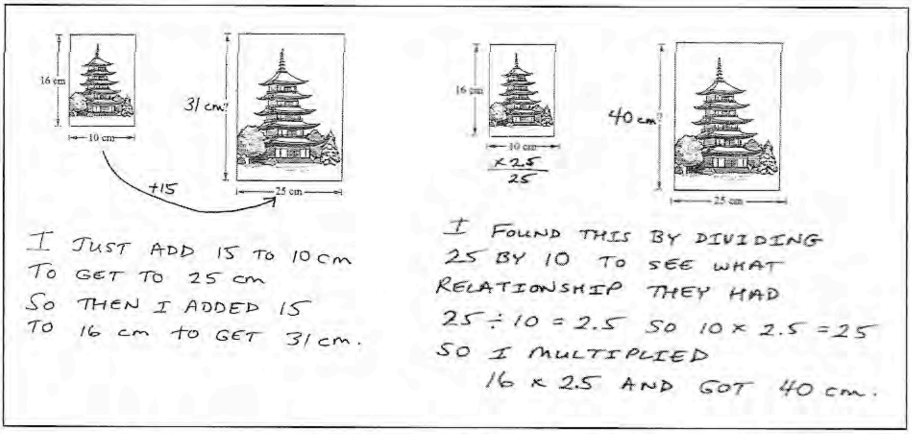

Fig. 18.5. Poster task examples used to introduce Willard research lesson (Task provided by the Mathematics Assessment Collaborative, Noyce Foundation)

Teachers in other districts, as well, were applying re-engagement to various mathematical topics. Many teachers had seen the re-engagement strategy demonstrated in public research lessons taught during annual meetings of the regional lesson study network. For example, SMFCSD teacher Mareva Godfrey (who observed Fisher's group at the January 2008 regional network meeting) tried out re-engagement in her teaching back in her own classroom. She wanted to try it because:

> Typically, our math ... program asks us to invite students to come up and share different algorithms they used, *but for the same answer, the correct one* [emphasis added]. This [using re-engagement] gave me an opportunity to look through the student papers [after the lesson], group answers, whether right or wrong, and look for patterns in misconceptions. Then the students addressed the misconceptions through the discussion. Of course, the correct answer and the different ways of solving the problem were also discussed.

She noted that the re-engagement strategy gave her time between lessons to analyze student work, identify important misunderstandings, and rewrite solutions in her own writing:

> Even though some of the kids recognized their particular answers and algorithms, because the charts up for discussion were in my own writing, [students] seemed emotionally distanced from what I presented and were very engaged in solving the mystery "What was this student thinking?"

Godfrey brought the re-engagement strategy to her lesson study group within SMFCSD, which added it to a subsequent research lesson on bar graphing. Godfrey and teammates also gathered data on the impact of the re-engagement strategy. When students were handed back their own papers, after the re-engagement phase of the lesson, twenty-eight of the thirty students in the class changed their numerical answer and/or added to their explanation, suggesting that they had improved understanding.

The re-engagement strategy also spread to Emery Unified School District (EUSD) through coaches in the SVMI network. Four teachers at Anna Yates School experimented with re-engagement in a wide range of lessons (regular classroom lessons as well as research lessons), using re-engagement to compare strategies to subtract multidigit numbers, to consider what constitutes a good mathematical argument, to revisit responses to difficult open-ended assessment items, and to explore various issues within language arts instruction. Anna Yates teachers observed students during research lessons to see whether and how students used the information from the re-engagement phase of the lesson to revise their own work during the latter part of the lesson, data which convinced them of the useful "dynamic that happens when students look at their own work and that of friends. . . . It's easier for them to get it when other students say it."

In summary, educators in lesson study groups across the San Francisco-Silicon Valley region actively developed, refined, and spread a solution to a persistent problem of practice—how to make student thinking visible and available for discussion. Teachers adapted re-engagement to different topics, different grade levels, and different purposes (such as making links between solution strategies or analyzing misunderstandings). Teachers observed students and studied their work to see whether and how the "re-engagement" phase of the lesson affected students' development of the target mathematical ideas, for example, whether students could explain how additive and multiplicative thinking would play out in a new problem. Practitioners took initiative in using and spreading re-engagement, building in useful features (such as recopying student work in the teacher's handwriting, collecting data on how many students revised their thinking, and pre-planning the choice of student work and flow of ideas around it). Re-engagement enabled teachers to use, within the rapid flow of classroom instruction, knowledge about student thinking and mathematics developed in professional development settings outside the classroom. Re-engagement also spread from mathematics to language arts and from the SVMI network to other lesson study groups.

The Context for Development and Spread of Re-engagement

The development and spread of re-engagement illustrates the potential of a regional professional learning community in which teachers take substantial, ongoing initiative to improve practice through design, teaching, and analysis of research lessons. Foundation personnel, funds, and technical assistance provided an infrastructure for professional learning that included opportunities for lesson study (within and across districts) as well as other opportunities to learn about mathematics and its teaching (such as coding of the MARS assessment). However, this is not a case in which teachers implemented reforms defined by outsiders. Rather, teachers actively worked together and drew on many kinds of knowledge resources (including one another's practice) to develop, refine, and disseminate their collective knowledge about mathematics teaching.

What conditions allowed the development and spread of knowledge about teaching in this region? Four aspects of the case are noteworthy.

1. Teachers had access to high-quality mathematical and pedagogical knowledge through SVMI. Recall, for example, how members of the SVMI's coaches lesson study group worked with Fisher to "push on the mathematics" of proportional reasoning, enabling them to identify the specific student thinking to be noticed and developed during lessons.
2. Lesson study allowed *practice-based* sharing and development of knowledge. Teachers could see what colleagues meant by ideas like "eliciting student thinking," providing a common referent for development of knowledge about teaching. In an early lesson study group with coaches, Fisher was quite surprised to see how her teammates implemented teaching strategies discussed by the team, leading her to remark on "the difference between talking about an idea or strategy and seeing it in action." Lesson study enabled teachers to gain knowledge about their students' mathematical thinking during lessons, and also to bring knowledge from their mathematics professional learning outside the

classroom (such as scoring MARS assessments) into live classroom instruction. (It also allowed SVMI personnel to see how ideas from professional development were translated into classroom instruction, providing valuable formative feedback on SVMI's professional development initiatives.)

3. Lesson study and daily practice were both *ongoing,* so that teachers could progressively refine their ideas through repeated trials in practice. For example, Willard teachers initially found that some students did not take advantage of the ideas summarized on the board (see fig. 18.4), and this prompted teachers, during the next lesson study cycle, to preplan how student ideas would be presented visually and selected for discussion.
4. A strong *collaborative* infrastructure was provided by SVMI and by lesson study. SVMI personnel collaborated on lesson study teams and put their own teaching out for observation and discussion during research lessons, creating an unusually level playing field for learning between the providers and consumers of professional development. Lesson study protocols and norms emphasized collaborative inquiry into student thinking rather than evaluation of one another as teachers (Lewis 2002). Lesson study teams provided opportunities to collaborate with, learn from, and be challenged by colleagues from other schools and districts, thereby allowing teachers to become members of a broader professional community dedicated to instructional improvement. Membership in a valued community provides an important source of motivation and identity (Solomon et al. 2000; Wenger 1999), and participation in lesson study can increase the perceived effectiveness of learning with colleagues (Perry et al. 2009). The close observation of students at the heart of lesson study is also likely to build teachers' motivation. Prior research has shown that students are a powerful motivator of teachers' improvement efforts (McLaughlin and Talbert 1993), and lesson study enables teachers to see instruction through the eyes of students as well as through the diverse lenses of their colleagues.

Teachers' leadership in building knowledge was supported, sustained, and amplified, we posit, through "virtuous cycles" in which teachers experienced greater efficacy in the classroom as they used strategies like re-engagement, stronger identification as members of a professional community devoted to improving practice, and increased motivation to further build and spread knowledge.

Working collaboratively to solve problems of teaching, particularly under conditions of good access to mathematical and pedagogical knowledge, may help teachers strengthen both their collective sense of agency and their participation and identification with a community of practitioners dedicated to improvement of instruction.

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REFERENCES

Ball, Deborah L., and Hyman Bass. "Making Believe: The Collective Construction of Public Mathematical Knowledge in the Elementary Classroom." In *Yearbook of the National Society for the Study of Education, Constructivism in Education,* edited by D. C. Phillips. Chicago: University of Chicago Press, 2000.

Boyd, Sally E., Eric R. Banilower, Joan D. Pasley, and Iris R. Weiss. *Progress and Pitfalls: A Cross-Site Look at Local Systemic Change through Teacher Enhancement.* Chapel Hill, N.C.: Horizon Research, Inc., 2003.

Fernandez, Clea, and Makoto Yoshida. *Lesson Study: A Japanese Approach to Improving Instruction through School-Based Teacher Development.* Mahwah, N.J.: Lawrence Erlbaum Associates, 2004.

Fisher, Linda, and Sally Keyes. "We Have What You Want: High-Quality Mathematics Tasks That Improve Learning." Powerpoint presented at Imagine: Mathematics Assessment for Learning (A Convening of Practitioners and Partners) Conference (Session III), Chicago, Bill & Melinda Gates Foundation, 2009.

Fisler, Jennifer L., and William A. Firestone. "Teacher Learning in a School-University Partnership: Exploring the Role of Social Trust and Teaching Efficacy Beliefs." *Teachers College Record* 108, no. 6 (June 2006): 1155–85.

Foster, David, and Audrey Poppers. *Using Formative Assessment to Drive Learning. The Silicon Valley Mathematics Initiative: A Twelve-Year Research and Development Project.* Palo Alto, Calif.: The Noyce Foundation, 2009.

Gorman, Jane, June Mark, and Johannah Nikula. *A Mathematics Leader's Guide to Lesson Study in Practice.* Portsmouth, N.H.: Heinemann, 2010.

Kenney, Patricia A., Mary M. Lindquist, and Cristina L. Heffernan. "Butterflies and Caterpillars: Multiplicative and Proportional Reasoning in the Early Grades." In *Making Sense of Fractions, Ratios, and Proportions,* 2002 NCTM Yearbook, edited by Bonnie Litwiller and George Bright, pp. 87–99. Reston, Va.: NCTM, 2002.

Lesh, Richard A., Mark Hoover, Bonnie Hole, Anthony E. Kelly, and Thomas R. Post. "Principles for Developing Thought-Revealing Activities for Students and Teachers." In *Handbook of Research Design in Mathematics and Science Education*, edited by Anthony E. Kelly and Richard A. Lesh, pp. 591–646. Mahwah, N.J.: Lawrence Erlbaum Associates, 2000.

Lewis, Catherine. *Lesson Study: A Handbook of Teacher-Led Instructional Change.* Philadelphia: Research for Better Schools, 2002.

Lewis, Catherine, Rebecca Perry, and Jackie Hurd. "Improving Mathematics Instruction through Lesson Study: A Theoretical Model and North American Case." *Journal of Mathematics Teacher Education* 12, no. 4 (August 2009): 285–304.

Lewis, Catherine, Rebecca Perry, Jackie Hurd, and Mary Pat O'Connell. "Lesson Study Comes of Age in North America." *Phi Delta Kappan* 88, no. 4 (December 2006): 273–81.

Lewis, Catherine, and Ineko Tsuchida. "Planned Educational Change in Japan: The Case of Elementary Science Instruction." *Journal Of Educational Policy* 12, no. 5 (September 1997): 313–31.

Lo, Jane-Jane, Tad Watanabe, and Jinfa Cai. "Developing Ratio Concepts: An Asian Perspective." *Mathematics Teaching in Middle School* 9, no. 7 (March 2004): 362–67.

McLaughlin, Milbrey W., and Joan E. Talbert. *Contexts That Matter for Teaching and Learning: Strategic Opportunities for Meeting the Nation's Educational Goals.* Stanford University: Center for Research on the Context of Secondary School Teaching. Original edition. Stanford University, 1993.

Mills College Lesson Study Group. *Can You Find the Area? Three Mathematics Research Lessons.* Oakland, Calif. DVD, 2003.

Perry, Rebecca, and Catherine Lewis. "Building Demand for Research through Lesson Study." In *Research and Practice in Education: Building Alliances, Bridging the Divide,* edited by Cynthia E. Coburn and Mary K. Stein. Lanham, Md.: Rowman & Littlefield, 2010.

Perry, Rebecca, Catherine Lewis, Shelley Friedkin, and Elizabeth Baker. "Teachers' Knowledge Development during Lesson Study: Impact of Toolkit-Supported Lesson Study on Teachers' Knowledge of Mathematics for Teaching." Paper presented at the Annual Meeting of the American Educational Research Association, San Diego, March 2009.

Singapore Ministry of Education. *Primary Mathematics 5A, 6A. Teacher's Guide.* Ministry of Education, Curriculum Planning and Development Division. Singapore: Federal Publications, 1994.

Solomon, Daniel, Victor Battistich, Marilyn Watson, Eric Schaps, and Catherine Lewis. "A Six-District Study of Educational Change: Direct and Mediated Effects of the Child Development Project." *Social Psychology of Education* 4, no. 1 (March 2000): 3–51.

Takahashi, Akihiko. "Beyond Show and Tell: Neriage for Teaching through Problem-Solving: Ideas from Japanese Problem-Solving Approaches for Teaching Mathematics." Paper presented at the Annual Conference of the International Congress for the Psychology of Mathematics Education, Mexico City, July 2008.

Wang-Iverson, Patsy, and Makoto Yoshida. *Building Our Understanding of Lesson Study.* Philadelphia: Research for Better Schools, 2005.

Wenger, Etienne. *Communities of Practice: Learning, Meaning, and Identity.* New York: Cambridge University Press, 1999.