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For most teachers, learning to translate the abstract principles that underlie effective mathematics instruction into competent management of the contingent, moment-to-moment character of classroom process is fraught both intellectually and emotionally. Developing Mathematical Ideas (DMI) is a curriculum for K-8 teacher learning designed in response to this challenge. It uses case discussion to model how close attention to student thinking and a deep grasp of the mathematics at issue can lead teachers to a more coherent, hence more confident, approach to their instruction. DMI consists of seven modules, each focused on a particular mathematical theme studied over eight 3-hour seminar sessions. It integrates case discussion with opportunities for participants to explore relevant mathematics, investigate the mathematical thinking of their students, analyze lessons from innovative curricula, and consider related research from the education literature. This paper excerpts the discussion of a single case from the second session of a seminar, *Reasoning Algebraically About Operations*, to demonstrate how teachers can come to recognize that questions about pedagogical strategy should be *based on* and *follow from* consideration of the mathematics content and student thinking.

Developing Mathematical Ideas: A Program for Teacher Learning[1]

By Deborah Schifter and Virginia Bastable

When, in 1993, the authors of this paper, together with Susan Jo Russell, began a teacher enhancement project called Teaching to the Big Ideas (Schifter, et al., 1999), we little imagined that we were embarking on a 14-year-long journey that would eventuate in a case-based professional development curriculum. Our goals then were broadly consistent with the NCTM Standards (1989, 1991) and have remained so throughout. Originally, we planned to investigate, in collaboration with 36 teachers, the principal conceptual issues encountered in grades K-8 as teachers center their practice on their students' mathematical thinking. We intended to identify those conceptual issues, to see how they arise and are treated in different classroom settings, and to follow their development through the grades.

In order to pursue this inquiry—and also as a professional development exercise—once each

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month participating teachers were to record a mathematics discussion from their classrooms and write narrative accounts, including verbatim dialog, based on those discussions. The professional development purpose of this task was actually articulated by teachers who had been given a similar assignment in a previous project (Schifter, 1996a, 1996b). They explained that having had to record the very words their students used made them attend to their students' ideas differently. Many confessed that, when writing up their narratives, they had realized that in the moment they had misinterpreted those words. They also reported that over time, as students became aware that their teachers were listening to them more carefully, their students became more thoughtful.

When we began the Teaching to the Big Ideas project, we thought of these records of classroom discussion as "raw" data, to be used internally by project staff and participating teachers. But having collected several rounds of episodes, and as we began to group them according to topic, we could see that they might profitably be used by other teachers as the basis of case discussions. Soon we began to envision casebooks, each casebook addressing a central mathematical theme, and organized into chapters, each one focused on a sub-topic. The resulting collection of cases would illustrate how related mathematical ideas arise in different classroom contexts and develop over the grades.

Studying how others had written about the use of cases in teacher education (e.g., Barnett, 1991; Merseth, 1995; Shulman, J., 1992; Shulman, L., 1986; Sykes & Bird, 1992) and experimenting with our own sets, we saw how case discussion offered teachers learning opportunities consistent with our vision of effective mathematics instruction. As with other areas that center professional development on the case method (e.g., business, law, medicine), teaching is both a very complex and a context-dependent activity. The principles that, in our view, underlie effective teaching practices are such abstract directives as: know the mathematics; center your lesson on student thinking; try to discern a logic in students' mistakes; and so on. For most teachers, learning to translate these into competent management of the contingent, moment-to-moment character of classroom interaction is a process fraught both intellectually and emotionally. For example, in our experience, as teachers begin to enact a practice in which student talk is foregrounded, they invariably seek reassurance and ask to be given lists of "good" questions. However, questions out of context are unlikely to be useful. By studying cases, that is, teachers can learn that there is no such thing as a "good" question in isolation from the flow of conversation. Instead, they come to see that effective instruction depends upon a deep grasp of the mathematical goals of the lesson, and bringing students from where they are to the mathematics to be learned requires careful attention to their thinking. Case discussions can model how such attention to student ideas coupled with the ability to identify significant mathematics can help teachers develop a feel for questions that can open up important areas for exploration. In short, though "right" questions cannot be anticipated, teachers can learn to approach their mathematics instruction with a coherent set of priorities.

We also recognized that other forms of professional development activity provide their own learning opportunities. Thus, we still wanted teachers to capture their students' mathematical conversations in writing, as well as analyze lessons laid out in innovative K-8 curricula, read about studies in the mathematics education research literature, and do mathematics for themselves. Especially, we wanted teachers to engage in mathematical explorations that allowed them to experience the challenge—and satisfaction—of understanding content that had previously eluded them.

Furthermore, we saw the benefits of coordinating these professional development activities. Thus, teachers' own mathematical explorations would position them better to interpret the mathematics students in the cases were working on. At the same time, a set of cases would allow teachers to see how the mathematics they were learning can arise in different contexts with students of different ages. Then, having studied a mathematical issue addressed in several cases, they could investigate how their own students engaged that idea. Each activity could enhance the others.

Eventually, these considerations led to the production of a professional development series called Developing Mathematical Ideas (DMI). DMI consists of seven modules[2], each module focused on a particular mathematical theme studied over eight 3-hour sessions. Facilitator guides include mathematics activities, focus questions, and assignments for each seminar meeting.

Although the richness of DMI comes from the coordination of a variety of activities and connections among concepts studied over time, in this paper we focus on a single case. We shall see how the case discussion provides opportunities for teachers at once to work on a mathematical issue, examine student thinking, and consider how one teacher's pedagogical strategies respond to her students' ideas. To begin, consider the following case from a kindergarten classroom.

A Teaching Episode[3]

The teacher, Lola[4], has set up her students to play Double Compare[5], a card game similar to War: Each card bears a numeral, from 1 to 6, and a picture of that number of objects. Players lay down the top two cards from their piles, and the player with the higher total when the numbers on the cards are combined says "me." For example, when Wei turns over a 2 and a 6, and Marta turns over

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[2] The seven DMI modules, published by Dale Seymour, Pearson Learning Group are:

- *Building a System of Tens*
- *Making Meaning for Operations*
- *Examining Features of Shape*
- *Measuring Space in One, Two, and Three Dimensions*
- *Working with Data*
- *Reasoning Algebraically About Operations*
- *Patterns, Functions, and Change*

[3] The episode is excerpted from "Double Compare," which appears in *Reasoning Algebraically about Operations, Casebook*, Schifter, et al., 2008, pp. 31-33.

[4] Pseudonyms are used for teacher and students.

[5] The game, Double Compare, is taken from *How Many in All?*, a kindergarten unit of *Investigations in Number, Data, and Space* (TERC, 1998).

a 3 and a 4, they count up the totals and Wei says "me." Lola describes classroom events as the game gets underway.

As I watched, a situation came up with several groups: each partner would have one card equal to the other person's and one card that was different. When Martina had 6 and 2 and Karen had 6 and 1, Karen quickly said "You." I asked how she knew and she pointed to the 2 and said, "This is big. Even though these are the same [the 6s], this [the 6 and 2] must be more."

Paul and his partner had a very similar set of hands. Paul put down 6 and 3 and his partner put down 6 and 1. Paul said, "I had 6 and he had 6, and then I had a higher number." I asked what their cards added up to, and both of them counted all the little pictures on their cards to get the totals. As they continued to play they did not count to figure out totals or who had more, but did accurately figure out who got to say "me." On a turn a minute later, Paul had 4 and 3 and his partner had 4 and 5. Paul's comment was, "I have 3 and he has 5." He knew he could basically ignore the two 4s. Another hand I saw was when Karen had 6 and 5 and Martina had 0 and 2. Karen said "Me, because she got two low numbers."

After a while, I realized that it seemed almost no one was EVER adding or counting or figuring out totals. I looked around some more, mostly just collecting data mentally about whether I saw adding, counting, or discussion of totals. I saw just a tiny little bit.

It was time to clean up and meet on the rug. Once we were all settled, we talked about how students knew who got to say "me." We talked for a while about how all the pairs "ignored" cards when each partner had the same one, and only paid attention to the cards that were different. Martina said that 6 and 3 is more than 6 and 1 because the 3 is bigger than the 1. I asked, "What about the sixes?" and she said, "They're the same." Paul added, "They don't matter. You don't have to pay attention to the sixes." I pointed out to them that when I put down the 6 and 1, they said, "That's seven," but when I put down the 6 and 3, no one figured out what it made. "Would 6 and 3 make a higher number than 6 and 1?" I heard 8! 9! 10! They settled on 9 by counting all, and because Danielle said 6 plus 3 is 9. "Is 9 more than 7?" Yes!

These students seem to have made a generalization, that a number plus a big number is more than the same number plus a small number. I put out a few more sets of cards, varying the number that was the same ("Does this only work for 6?" "No.") They said it always works, and Paul reiterated that you don't have to pay attention to the numbers that are the same.

Another generalization most of them seemed to be using was something about two small numbers is less than two big numbers. Karen's comment that she got two low numbers expressed this idea. I asked the group about this. I put out 1 and 5 and 0 and 4, which had been a turn in Amanda and Danielle's game. I asked how Amanda knew she had more. She said, "This [5] is bigger than this [4], and this [1] is bigger than this [0]." I asked if it would work with other numbers and everyone said yes. We tried some. They were all saying it worked. They weren't adding and counting. They were "just knowing."

Implicit in the children's actions were two generalizations. For one, the children were close to articulating what it was: "You don't have to pay attention to the sixes." I wonder what it will take for them to have words for their second generalization beyond simply saying they "just knew."

Before moving to the case discussion, let us consider what this case illustrates. In particular, consider how actions of the teacher, Lola, are based in her knowledge of mathematics and her attention to student thinking.

Double Compare was designed to give young children an opportunity to practice counting up to find totals. Yet, as they begin to play the game, it seems some of Lola's students are subverting that goal of the exercise: They play many rounds correctly, but without totaling.

However, Lola quickly gets beyond noticing what her students *aren't* doing, to figure out what they *are* doing. Paying careful attention to their strategies, she realizes that implicit in their moves is a general principle: If each child has a card of equal value, compare the other two cards; the child with the card of greater value has the greater total. When it is time to put the cards away, Lola brings her students together to discuss this idea.

Lola has a sense of the important role of generalization in the doing of mathematics—and she recognizes that even kindergartners can participate in such a central mathematical activity. As young children learn about numbers and operations, they begin to see regularities in our number system, and to think about what stays the same among things that are changing. The regularity, or generalization, illustrated above—if one number is greater than another, and the same number is added to each, the first total will be greater than the second—is one example. This statement is true for *any* three numbers. For example, since $54 > 36$, $54 + 98 > 36 + 98$. This idea can be expressed in algebraic notation: *For all numbers a, b, and c, if* $a > b$, *then* $a + c > b + c$.

However, it is important not to attribute too much understanding to these kindergartners. The children are playing a card game that involves the numbers 1 through 6. They determine who gets to say "me" by combining the number of pictured objects when they each turn over two cards. We do not know if the children have a conception of numbers greater than 6, whether they think in terms of the operation of addition, or whether the ideas they are discussing apply to contexts outside the card game.

Yet as this episode illustrates, teachers can invite even very young children to talk about the general ideas implicit in their actions. Consider Lola's moves: First, she notices that some of the children are playing the game in a way that indicates they may be acting on a general principle or rule which they notice but have not articulated. Next, she decides to bring this to the attention of the whole class during discussion. Then she asks her students to explain their rule and why it works by suggesting additional examples for them to consider. Finally, she poses the question: Does this rule work for just a few examples or does it apply more generally?

That is, having paid careful attention to the ideas her students are engaging, Lola is able to situate her students' actions in the context of important mathematics. By pressing them to explain themselves as they work in pairs, and then raising questions for whole-group discussion, she brings their attention to the mathematical generalization implicit in their activity.

A Sample Case Discussion

The following excerpt from a DMI seminar illustrates how case discussion can provide a context for teachers to deepen their mathematical understanding, sharpen their focus on student thinking, and examine the pedagogical moves that follow from these. Participants in the seminar are practicing teachers of first grade to middle school. In this excerpt, the teachers are discussing the kindergarten case, "Double Compare," presented above. It takes place in the second session of the seminar, *Reasoning Algebraically About Operations*.

Deepening mathematical understanding: Expressing generalizations

One goal of the *Reasoning Algebraically About Operations* seminar is to learn to identify and articulate mathematical generalizations in both natural language and algebraic notation. This, in turn, serves several purposes: 1) teachers will be better prepared to notice when their students broach important mathematical themes, 2) they can identify different levels of generality and consider the extent of the generalization their students may be formulating, 3) they can learn the conventions of algebraic notation, 4) they can vest algebraic notation with meaning, since they are using the notation to formulate ideas already articulated, and 5) they can learn to appreciate the conciseness of that notation.

To see how teachers begin to take initial steps toward these goals, consider the following segment of the case discussion. The facilitator begins by asking for the generalizations Lola's students were working with.

Evelyn speaks up first, "This is what I wrote for Paul. He is talking about the cards he and his partner have, 4 and 3 versus 4 and 5. I wrote this for what Paul did. If two cards are the same, just ignore them and compare the other two numbers."

Lucy agrees, "That was on my list of generalizations, too."

In order to challenge participants to produce a more precise statement, the facilitator suggests a set of cards that satisfies Evelyn's condition, but is not a situation she intends to include: "How about this situation, 2 and 3 versus 5 and 5? Would that apply? It matches the words Evelyn used, 'If two cards are the same, just ignore them.'"

Evelyn refines her statement, "You have to say, if each of us has a card that is the same, then look at the cards we have that are different."

Beverly refers back to Lola's language in the case and says, "I think there is another way to say it. It seems like the same idea but also a bit different. A number plus a big number is more than the same number plus a smaller number. It might be easier with some letters."

The facilitator proposes that the group consider another set of cards: "If one person has 5 and 3, and the other person has 5 and 2, which is the big number?"

Beverly sees her point, "Oh, yeah. In that case, 3 is the big number, even though it's smaller than 5."

Evelyn adds, "I tried symbols, too. It was odd because it was almost easier with symbols." She comes to the board and writes, "$n + a > n + b$ when $a > b$."

Because Evelyn has just introduced an idea related to one of the seminar's central goals, the facilitator turns to the group and asks, "What made it seem easier?"

Naomi: "You can tell what 'bigger number' refers to, and you can say '*n*' for the same number without having to write it all out."

Then Nancy brings the group's attention to another part of the case, "I think there is another generalization here. How about this: Two small numbers added together is less than two big numbers added together."

The facilitator decides not to pursue a more precise statement in English and asks instead if anyone has written a version of Nancy's words in algebraic notation.

Linda comes to the board and writes, "*If* $a < b$ and $c < d$, then $a + c < b + d$." She then turns around and says, "But I am wondering, in the card game, the numbers are from 1 to 6. Do we need to say that too?"

Linda has raised a significant issue: To what domain does the generalization apply? And in the context of this seminar, the question has two forms. 1) What is the domain the students are considering? 2) In what domain is the generalization true? To conclude this portion of the discussion, the facilitator says, "Linda has made an important point. If we assume these statements are based on the context of the game in the case, then the variables are the whole numbers from 1 to 6, but are the statements true more generally? What are the numbers we can choose for *a, b, c* and *d* and still have the statement be true? When we write statements in symbols, we need to consider the domain. We will look into this more as the seminar goes on."

In the first part of the case discussion, participants share both symbolic and natural-language versions of the generalizations they uncover in the thinking of Lola's kindergartners. They use the notation so their statements accurately reflect their ideas. In addition, participants note that symbolic statements are sometimes easier to write and understand than the natural-language versions. Finally, they begin a conversation about the domain the variables satisfy, something that will be examined in more detail as the seminar continues.

Focusing on student thinking: Are students thinking about operations?

The seminar discussion presented in the previous section shows that even as participants engage the mathematics, they are also paying careful attention to student thinking, studying the kindergartners' words in order to uncover the general principles underlying their strategies. When Linda asks her question about the domain, the facilitator uses the opportunity to highlight the boundary between the students' ideas (they might be thinking only in terms of the numbers 1, 2, 3, 4, 5, and 6) and the more general mathematical principles the teachers are sorting out for themselves (the generalization is true for all real numbers).

As discussion continues, the teachers extend the generalization in another direction—now to consider other operations—and then turn back to the kindergartners to consider more specifically just what mathematics they are working on.

Kathleen begins this part of the discussion by comparing Lola's case to the work of her third graders: "You also need to think about what operation this works for. My class is working on arrays and there's a game where they compare array cards. I hear kids saying something very similar, 'I know if this is 6 x 5 and 6 x 8, I can just ignore the 6s and compare the other numbers.' When I asked them to explain, they say, 'You have 8 rows of 6 chairs or 5 rows of 6 chairs. All you need to do is see how many rows you have and then you are all set.' So then the question is, in which operations can you do this? When can you ignore one of the numbers and just compare the others? How do the kids get to know that?"

Kathleen raises an important issue. She points out that her third graders are working on an analogous problem. Adapting Evelyn's formulation in algebraic notation, one might represent the generalization implicit in the third-graders' explanation as, "For all counting numbers n, a, and b, $n \times a > n \times b$ when $a > b$." (Note that the context of the third graders' thinking is arrays, which necessarily involve counting numbers.)

But Kathleen has asked, when students make such generalizations, are they thinking in terms of a specific operation—about joining two sets (in the case of the kindergartners), or about multiplying (in the case of the third graders)? Or are the students thinking that under any operation, if one number of each pair is the same, you can compare the remaining numbers?

Later in the seminar session, participants will turn to another case, one of fourth graders who have come across the same question with regard to subtraction[6]. Comparing 145-100 and 145-98, it seems to them counterintuitive that 145-98 gives the *larger* result, when 98 is *less than* 100.

Knowing that the teachers will have an opportunity to discuss the subtraction case, the facilitator underscores Kathleen's point while also suggesting the group will return to this issue: " The questions Kathleen is asking are important ones for us to keep in mind. Once a generalization is noticed in one context or with one operation, what happens if the operation changes? Will it still be true? Why or why not? As students work on such questions, they learn more about the operations. We can, too. Let's be sure to mark these questions so we can come back to them."

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[6] "Is it two more or two less?" in *Reasoning Algebraically About Operations*, Schifter, et al., 2008, pp. 45-47.

Linda brings the conversation back to the kindergartners, pointing out that, in contrast to Kathleen's third graders, Lola's students can't consider other operations yet: "Since this is a kindergarten class, they only have counting and maybe adding."

Nancy says, "But they're just looking at the cards. It doesn't seem as much like addition."

Evelyn disagrees, "It is because they know they are putting amounts together they are saying you can ignore the part that's the same. If they weren't adding them, then what are they doing? They know they don't have to actually find the total to know which combined amount is greater. The whole context is addition."

But Beverly objects, "I'm not sure if the children are really thinking about addition. In line 39, Amanda just seems to be matching and comparing. Amanda says something like, 'This is bigger than this and this is bigger than this.' It feels like she is just looking at the cards and not doing any adding."

Lucy expresses bewilderment, "In the case, Lola was saying they weren't adding or counting. They were 'just knowing.' I keep going back and forth myself. What are they doing? Counting? Comparing? Adding?"

Kathleen responds, "I think Lola's point was that they didn't have to find the total in order to compare. So in a sense, they weren't counting or adding. But they did seem to understand that they were asked to compare the results of a combining situation."

The question participants are debating isn't resolved. Instead the facilitator asks, "Why is this a useful question to consider? How does it help us understand what is important for students to learn?"

There is a pause in the discussion. In fact, it isn't necessary for participants to answer the question at this time. But it is an important question to sit with: Why does it matter whether the kindergartners are thinking in terms of the operation itself or are just thinking about cards. Or, more generally, why should teachers work so hard to understand the thinking of these students?

Instead, Naomi contrasts the discussion with her own practice, "It is really odd for me to think in so much detail. With my middle schoolers, I am not used to noticing the kinds of differences Lucy is talking about."

Rather than answer the question herself, the facilitator encourages Naomi to take on this task: "Listening closely to student ideas is something to explore throughout this seminar. For each session, the cases offer you opportunities to think about students' ideas in detail. But you are also asked to do assignments to record the mathematical conversations that take place in your own classroom. This is an opportunity for you to examine the ideas of *your* students."

Investigating teacher moves: Guided by mathematical understanding and attention to students' ideas

The normative mode of professional sharing in current school culture involves teachers showing one another "activities that work," and analysis rarely moves beyond superficial and evaluative reactions: "I like what she did when . . ."; "Instead, she should have . . . ." In the initial meetings of a DMI seminar, the facilitator must help participants learn to examine the cases for what they reveal about mathematics and children's mathematical thinking: What is the mathematics at play in this lesson? Based on the evidence (what a child says, does, or writes), what might we hypothesize about what the student understands/does not understand? What is the mathematical idea he or she seems to be working on? Questions about pedagogical strategy are *based on* and *follow from* consideration of the mathematics content and student understanding.

As we shall see in the final excerpt from the discussion of the "Double Compare" case, the teachers in this seminar have moved beyond judging Lola's pedagogical actions, instead to consider how her moves grow out of careful attention to student thinking and, informed by her knowledge of the mathematics content, her goals for the lesson.

Nancy opens this part of the discussion, "The teacher let the children play the game their way. She doesn't stop them and make them count and add even if that is what she expects."

Evelyn continues, "From the beginning of the case, Lola looks around the room and sees that it isn't just a few students but most of the class is working this way. It feels like that is a moment when she is assessing what is going on and deciding what to do about it. Then she calls them all together to discuss it, doesn't she?"

Beverly adds, "She doesn't just observe what they are doing, but when she pulls them together she makes that the topic of conversation. She comes up with examples for them to look at that bring the idea out. She really thinks fast."

Wanting Beverly to articulate the idea Lola is working to bring out, the facilitator asks, " What is the idea you are talking about, Beverly?"

"That sometimes you don't need to add to determine who has the most."

Evelyn specifies, "Around lines 10 and 15, Lola seems to say the point of the game is about finding totals—counting and adding." She continues, "By the time she gets to the whole group discussion she has changed her objective. Now she wants them to be clear about how they could get the answer when they weren't adding."

Nancy elaborates Evelyn's point, "Lola notices they are not doing what she thought they would, but she also sees the value, the mathematical importance, of what they *are* doing. I mean she could have just thought, 'Well. This game isn't going very well. Let's put it away.' But she doesn't. She is sensing there is something else here."

As discussion continues, participants now consider how Lola conducts the whole-group discussion. But at this point, we have seen enough to form a good idea of what the teachers in the seminar are attending to. They see that, from the start of the Double Compare activity, Lola carefully observes her students as they play the game; that most students don't perform the task as expected; that she is quick to see the mathematical value in what they do do; and that subsequently Lola focuses whole-group discussion on the mathematical principles underlying her students' strategies.

CONCLUSION

The different chapters of this book illustrate a variety of approaches to case discussion. The use of cases in DMI is shaped by our view of effective mathematics teaching, our best guesses about the knowledge and skills required of teachers to enact such a practice, and a sense of how teachers might learn these.

Specifically, we start with a view of instruction that foregrounds student ideas along with clear goals for student learning. The art of teaching involves helping students move from where they are into the content to be learned. Such a practice depends on teachers having a deep understanding of mathematics content and the ability to situate students' ideas in that content.

For most teachers, learning to translate the abstract principles that underlie effective teaching practices into competent management of the contingent moment-to-moment character of classroom interactions is a process fraught both intellectually and emotionally. DMI's case methods respond to this challenge by modeling how careful attention to student thinking and a grasp of the mathematics at issue can help teachers develop a coherent approach to mathematics instruction.

DMI offers other forms of professional development activity to supplement the cases. Thus, each module integrates case discussion with opportunities for teachers to learn mathematics for themselves, analyze lessons from innovative curricula, study the mathematical thinking of their own students, and read about relevant research in the mathematics education literature. Modules are designed for eight 3-hour sessions, providing for sustained exploration of a major mathematical strand and illustrating central conceptual issues of the K-8 curriculum. Though this chapter describes and analyzes a discussion based on a single case, it is important to note that the context for this discussion is the second session of an eight-session seminar.

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